OSCILLOGRAPHS

A CONCISE TREATISE ON THE THEORY CONSTRUCTION, AND USE OF ELECTRO-MAGNETIC, HOT-WIRE, ELECTROSTATIC AND CATHODE RAY OSCILLOGRAPHS

BY

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THIS book is dedicated to my friend Mr. S. S. A. Watkins, my colleague for many years at the City and Guilds Engineering College, London. He gave me much valuable help on oscillographs, and rediscovered the method of damping oscillographs by means of a resonant shunt when working independently of me in America.

J. T. IRWIN.

BOVALLY,
LIMAVADY,
IRELAND.

PREFACE

ANYONE who worked at the Central Technical College, London (now the City and Guilds (Eng.) College, under Professor Ayrton, or later under Professor Mather, will understand the happy conditions which made possible the greater portion of the work represented by this book. While routine work had to be carried out with great care, every encouragement was given to work of an original character, and difficult problems were subjected to constructive and friendly criticism.

Equally happy were the relations between the junior members of the staff, and the author has to acknowledge the help received from his former colleagues and also from a long line of able students who assisted him with experiments. It would be almost impossible to make a complete list of these, and it would be unfair to those left out if an incomplete list were published. Special mention must, however, be made of the valuable help received from Mr. F. W. Andrews, the instrument maker at the College, and from Mr. J. King, the laboratory attendant.

With regard to the book itself the aim has been to make it suitable for any technical student of ordinary ability, as the mathematics do not go beyond the simplest integral and differential calculus, and the apparent complexity of some of the proofs is due only to a desire to put in all the steps necessary to the solution of the problem.

The only exception to this is the portion due to Mr. Hodgson and specially indicated in the text. This portion, which involves higher mathematics, can be skipped by those unable to follow the proof, and the results accepted so that there may be no break in the sequence.

The Cambridge Instrument Co. and Mr. R. W. Paul have been closely identified with oscillograph development in England, and the author has to acknowledge his indebtedness to these firms (now amalgamated as the Cambridge Instrument Co), for the loan of blocks referring to their products. Permission has also been kindly given to use Figs. 73–76, taken from Dufour's book on the cathode ray oscillograph.

The reading of the proofs was carried out by my friend and former student, Mr. R. E. Neale, in a very thorough manner, and many additions and explanations are due to him.

As the work is to a great extent original, and as the author has had no facilities for confirming some of the later theoretical results, he would be glad if anyone carrying out investigations along the lines mentioned would communicate the results to the editor or the author. He would also like to have any inaccuracies pointed out.

J. T. IRWIN.

CONTENTS

SYMBOLS AND ABBREVIATIONS

The following symbols and abbreviations adopted by the International Electrotechnical Commission are used in this volume—

A = ampere
mA = milliampere
μA = microampere
V = volt
μF = microfarad

ILLUSTRATIONS

TABLES

OSCILLOGRAPHS

CHAPTER I

FUNDAMENTAL PRINCIPLES

The Name and Invention of the Oscillograph. The name "oscillograph" was first adopted by Blondel to denote an instrument for indicating the instantaneous value of an electric current.

Almost any instrument which accurately indicated the current flowing in a circuit could be used to show the instantaneous value of an electric current if the latter changed very slowly from one value to another, but when the changes are extremely rapid a very restricted choice of instruments is available.

The oscillograph is concerned chiefly with the graphic representation of *very rapid* oscillations, and the name might perhaps have been chosen so as to indicate the chief application of the instrument.

The oscillograph, like so many other instruments, is a development springing from a need ; nevertheless, the work of Blondel was so outstanding, firstly in indicating the conditions to be fulfilled and secondly in perfecting an instrument, that it is only fair that he should be hailed as the inventor. On the other hand Duddell, starting with the bifilar type of instrument first suggested by Blondel, displayed so much skill and ingenuity in perfecting this type that it practically displaced the use of the moving iron type chiefly used up to that time by Blondel.

Other investigators have devised special oscillographs and the references given in the Bibliography (p. 162) should be consulted by those specially interested in the subject, as it is outside the scope of the present work to give anything in the nature of a detailed historical resumé.

Theory and Principle of Working. To study the theory and principle of working of an oscillograph it is best to take the simplest case and for this purpose the Einthoven galvanometer (or oscillograph) may be considered. The essential feature of this instrument is a stretched conducting fibre or wire placed in a strong magnetic field. The current to be indicated is sent along the wire and the position of the wire is indicated by some suitable means.

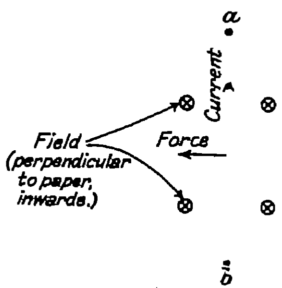


FIG. 1.—FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD.

If, in Fig. 1, *ab* is the stretched wire, and if it be assumed that there is a strong field across the wire from the observer into the plane of the paper, then there will be a force on the wire at every point tending to make it more towards the left when a current is flowing in the direction shown by the arrow, from *b* to *a*.

Every force, however, is balanced by an equal and opposite one, and, in the present case, the force due to the interaction of the conductor carrying the current with the magnetic field is balanced by the resultant of three other forces. These forces are—

1. The control force due to the tension in the wire. This acts to bring the wire back to the central position where the tension is least—that is a straight line between the points *a* and *b*. For small deflections the control force can be taken as proportional to the displacement of the wire from its central position.

2. The inertia force, equal to the mass times the acceleration. That is, the rate at which the velocity is changing multiplied by the mass gives the force in absolute units. If the velocity is increasing, the inertia force is opposing increase of velocity. If the velocity is decreasing, the inertia force is opposing the decrease in the velocity.

3. The frictional force due to the movement of the element through the surrounding medium. This force always acts in the opposite direction to which the wire, or the element of the wire, is moving, and its magnitude is roughly proportional to the velocity and to a coefficient which is determined by the fluid employed, the section of the wire and the nature of the surrounding enclosure of the wire.

From the consideration of the above rules it is evident that, if forces (2) and (3) could be made quite small compared with (1), then they could be neglected and we would have the one result that the force due to the interaction of the current and the magnet field would be opposed by the equal control force, but the control (or restoring force) acting on the wire is proportional to the displacement. Therefore, for this particular condition, the displacement would be proportional to the current flowing, and if this displacement could be observed or recorded, it would give a means of determining the strength of the current at every moment.

The greater the tension on the wire the larger is the force due to the control, and the more rapid the changes in the current which the wire is able to indicate accurately.

It will be seen later that an oscillograph without any added damping, apart from the natural damping of the wire itself, can indicate *very* accurately *periodic* alternating current of a frequency, one-tenth the natural frequency of the oscillograph.

On the other hand, no matter what the control force may be, if the rate of change of the current is very rapid the inertia forces will also be relatively large and will cease to be negligible. If there is a current switched on instantaneously then the wire will have a large velocity imparted to it and the kinetic energy stored in the wire by virtue of this velocity will be spent in backward and forward movements of the wire until the whole energy is dissipated, just as the impulse given to a pendulum is spent in backward and forward swings which gradually become less until the pendulum stops at the mid position.

Conversely, if there is a steady current flowing in the wire producing a steady deflection, and the current is suddenly stopped, the tension on the wire will give it a high velocity which will start free vibrations and the duration of these vibrations will depend only on the frictional forces which damp down the motion.

In both these last cases where there are free vibrations of the wire, apart from the original force that produced them, the displacement of the wire at any instant is no indication of any current value.

Such free vibrations should therefore not be allowed except where their occurrence cannot produce any wrong interpretation of the results and, generally speaking, oscillographs are used with a large frictional or damping force owing to the extremely rapid changes of current that can take place with electricity as compared with say, the change that could possibly take place in the cylinder of a steam or gas engine, where the indicator performs a function akin to that of the oscillograph for electricity.

Forces on a Current-carrying Wire Vibrating in a Magnetic Field. A simple wire, of mass m per cm. length, placed in a magnetic field will have the following forces, measured in dynes, acting on any unit length—

(1) The electromagnetic force $= \frac{i}{10} B$ dynes per centimetre length; where i is the current in amperes, and B is the strength of field in lines per square centimetre.

(2) The control force $= -\Delta T \frac{\pi^2}{l^2} = -k\Delta$; where Δ is the displacement of the particular length chosen from the central position of the wire. T is the tension of the wire in dynes, l is the length of wire between supports in cms.

(3) The damping force $= -\rho v$; where v is the velocity, in cms. per sec., and ρ is the coefficient that depends on the fluid employed, the section of the wire, and (to some extent) on the surrounding chamber.

(4) The inertia force $= -am$; where a is the acceleration in cms. per sec., and m the mass per unit length.

If the wire in Fig. 2 vibrates about its mean position PQ under the influence of an alternating current which varies sinusoidally, then its extreme amplitude is represented by the dotted lines in Fig. 2(a).

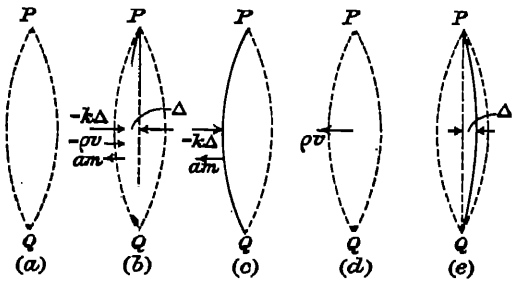


FIG. 2.—ILLUSTRATING CONDITIONS OF VIBRATION OF A STRETCHED WIRE PQ

If movements to the left and forces to the left be considered positive then, if the displacement of the wire in Fig. 2(b) is Δ and if *it be moving to the left*, the control force at that moment is $-k\Delta$, i.e. it is acting to bring the wire to rest or to stop the movement.

The frictional force is $-\rho v$, where v is the velocity, and also acts against the movement. The inertia force is acting in the direction of motion, tending to prevent change of motion, and is equal to the rate at which the velocity v is changing and, since the velocity is decreasing, the force is positive and is equal to $-(-am)$ or am. Therefore the inertia force is opposing the control force and, at that instant, the damping force is in the same direction as the control force.

When the wire reaches the extreme position to the left, shown in Fig. 2(*c*), the control force is still acting towards the right and has its maximum value as the displacement is a maximum. The damping is zero at that instant as the velocity is zero. The inertia force is a maximum, as the rate at which the velocity is changing is a maximum, and is acting in the same direction as in Fig. 2(*b*), i.e. in the opposite direction to the control force.

When the wire arrives on its return journey at the mean position, shown in Fig. 2(*d*), the velocity is a maximum and the rate at which the velocity is changing is zero, therefore the inertia force is also zero. As the wire is in the central position there is no restoring or control force on the wire, so the control is also zero; but, as the velocity is a maximum, the damping force is a maximum and now acts towards the left and is positive as the velocity is negative.

When the wire arrives at the position shown in Fig. 2(*e*), and is moving towards the right, the (negative) velocity is growing less and, as the force of inertia opposes the change of velocity, the inertia force acts towards the right and is therefore a negative force. The control or restoring force is towards the left and is therefore positive, while the damping force is also towards the left and positive as it tends to stop the movement.

If this reasoning be followed, step by step, it will be seen that for a wire vibrating under the influence of a simple periodic alternating current *the inertia force and the control force always act in opposite directions—that* *is, the control force is a maximum when the inertia force is a maximum and they are opposite in phase. To find their resultant it is only necessary to subtract the value of one from the other.*

On the other hand, the damping force is a maximum when the control force is zero and it can only be compounded with the control or with the inertia force vectorially to give effective values.

For instantaneous values, the forces can all be added algebraically thus, for the position shown in Fig. 2(*b*): If i is the instantaneous value of the current then—

$$\frac{i}{10}B - k\Delta - \rho v - am = o$$

since these are the only forces that can act on the wire and the sum of them must be zero at any moment.

If the changes taking place in the current be very slow then the terms ρv and am are very small and can be neglected when $\frac{i}{10}B = k\Delta$, and the deflection $\Delta = i/10k$. The condition that $\Delta = i/10k$ is only true for steady currents or for conditions where the rate of change is comparatively slow.

Graphic Representation of Conditions. Fig. 3 is a graphic representation of the cycle of events represented in Fig. 2. In Fig. 3(*a*) the sine curve marked Δ shows the displacement plotted against time. The curve marked $-k\Delta$ shows the control force acting at every point in the opposite direction to the displacement.

In Fig. 3(*b*) the curve marked v represents the rate of change of the displacement or the velocity, and $-\rho v$ is the damping or frictional force acting in the opposite irection to the move- ıent or velocity.

In Fig. 3(*c*), the ırve marked *a* shows ıe rate of change of elocity, and the curve .arked $-ma$ repre- ·nts the inertia force pposing the change velocity.

In Fig. 3(*d*), the ırve marked $-(k\Delta +$ ' $+ ma)$ represents e sum, at every in- ant, of the control, ımping, and inertia rces, and must have posed to it the equal rve $\frac{i}{10}B$ which re- esents the inter- tion of current and ld. The displace- nt curve Δ is also ıwn again for com- rison and it will be served that the dis- cement of the wire is ; a maximum when current is a maxi- m but lags (in this ticular case) about behind the current.

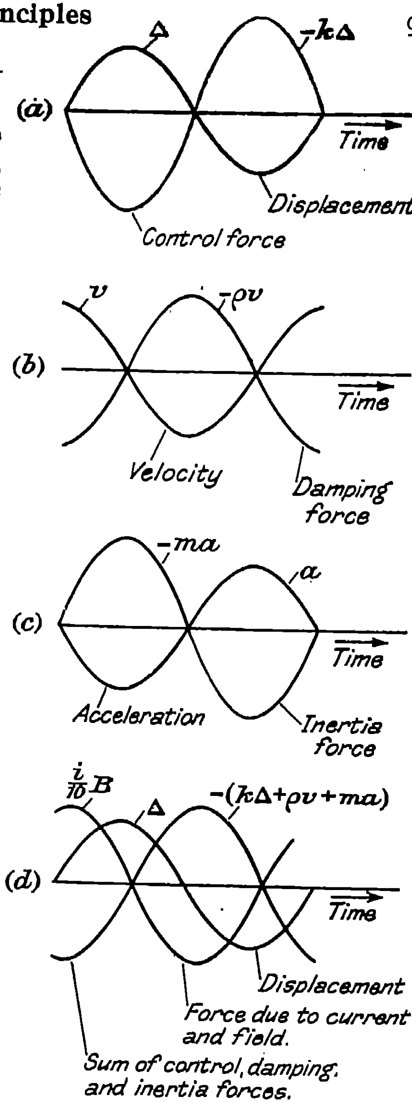


FIG. 3.—GRAPHIC REPRESENTATION OF CYCLE OF EVENTS SHOWN IN FIG. 2

Vectorial Representation of Conditions. A vectorial representation of the same conditions is given in Fig. 4.

If Δ be the maximum displacement, then the maximum velocity is $2\pi f\Delta$ where f is the frequency of vibration of the wire. Writing $2\pi f = \omega$, we have the maximum velocity $= \omega\Delta$ and leading by an angle $\frac{\pi}{2}$ on the displacement. The maximum acceleration is ω times the velocity and is $\omega^2\Delta$ and leads the velocity vector by $\frac{\pi}{2}$. The upper portion of Fig. 4 shows this condition.

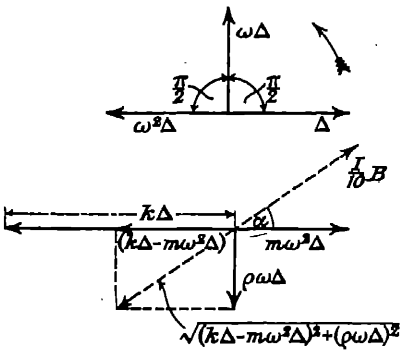


Fig. 4.—Vector Representation of Conditions Depicted in Fig. 2

The force diagram is shown in the lower portion of the figure where the control force is drawn in opposition to the displacement, the frictional force in opposition to the velocity, and the inertia force in opposition to the acceleration.

This diagram shows how the control force $k\Delta$ is always opposed by the inertia force $\omega^2\Delta m$ and how the difference between the forces $k\Delta$ and $\omega^2\Delta m$ compounded with the frictional force $\rho\omega\Delta$ gives the resultant maximum internal force equal to

$$\sqrt{(k\Delta - m\omega^2\Delta)^2 + (\rho\omega\Delta)^2};$$

and this force must be equal and opposite to the maximum external force $\frac{I}{10}B$. The lead of the current on the displacement is the angle α.

Effect of Frequency. It is evident from Figs. 3 and 4 that the resultant maximum internal force and therefore the value of the current will not be constant for a given deflection Δ but will vary with the frequency. The larger the control force, compared with the damping and inertia forces, the less the resultant internal force will vary with a given change of frequency, and therefore the less the change in the value of the alternating current to give an amplitude equal to Δ and the smaller the angle α between the current and the displacement.

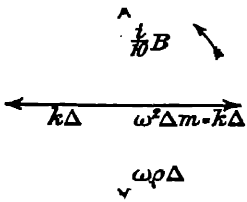


FIG. 5.—APPLIED FREQUENCY = RESONANT FREQUENCY OF THE WIRE; CONTROL FORCE = INERTIA FORCE; CURRENT LEADS BY $\pi/2$ ON THE DISPLACEMENT

If the frequency of the alternating current be raised until $\omega^2\Delta m = k\Delta$, then these two forces balance one another and do not require any external force to balance their resultant, and if the force $\rho\omega\Delta$ were made small the current required for a given deflection would be very small and the instrument would be a very sensitive detector of alternating current of that frequency. This explains the great sensitivity of the tuned vibration galvanometer.

When, however, $\omega^2\Delta m = k\Delta$ then $(2\pi f)^2 = k/m$

and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$;

but this is the resonant frequency of the wire, therefore, when the frequency of the alternating current is the same as the frequency of the wire, there is no internal force apart from the damping and the current leads by $\frac{\pi}{2}$ on the displacement. This is shown in Fig. 5.

If the applied frequency be raised *above* the resonant frequency of the wire then $\omega^2 m\Delta$ is greater than $k\Delta$,

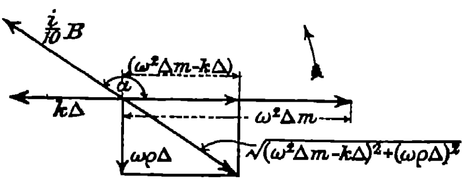


FIG. 6.—APPLIED FREQUENCY EXCEEDS RESONANT FREQUENCY OF THE WIRE; CURRENT LEADS BY MORE THAN $\pi/2$ ON THE DISPLACEMENT

and the condition of affairs is as shown in Fig. 6. The angle a between the current and the displacement is now greater than $\frac{\pi}{2}$ and approaches π as the frequency becomes higher.

Magnitude of the Frictional Force; Damping. So far nothing has been said about the magnitude of the frictional force as compared with the control force.

As the control force is independent of the frequency, and the frictional force varies with the frequency, the magnitude of the frictional force has to be defined at some definite frequency and from that the coefficient ρ deduced.

The maximum value of the frictional force is $\omega\rho\Delta$ where Δ is the maximum displacement.

If $\omega\rho\Delta$ were made equal to $k\Delta$ when $k\Delta = \omega^2 m\Delta$, then at this, the resonant frequency of the wire—

$$\omega\rho = k$$

$$\text{and } \rho = \frac{k}{2\pi f} = \frac{k}{2\pi\frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

$$\text{or } \rho^2 = km.$$

At the resonant frequency of the wires, since the control and inertia forces are equal and opposite, the only internal force is the frictional force which—at this frequency—has been made equal to the control force. This has to be balanced by the force due to the current in the wire, therefore, the current required will just have the same value as would be required to overcome the control force if the frictional and inertia forces were entirely absent, and the maximum displacement Δ due to a current $I \sin \omega$ will be equal to the steady deflection due to a current I. That is, the deflection for a very low frequency and for the resonant frequency of the wire will be the same.

This would correspond, in Fig. 4, to having all the vectors $k\Delta$, $m\omega^2\Delta$ and $\rho\omega\Delta$ of equal length at the resonant frequency of the wire.

This value of the frictional force has some merits in practice, and has been advocated as it gives practically the correct amplitude of deflection for moderately low frequencies, and for frequencies near the resonant frequency of the wire.

In general practice, however, the frictional force is made *twice as great*, i.e. $\rho = 2\sqrt{km} = 2\omega m$, so as to make the instrument what is known as "critically damped" or "dead beat." Under these conditions, for a sudden increase or decrease of the current, the deflection of the wire will just be increased or decreased in a corresponding degree, without any overshooting of the final value. Any decrease in frictional force below that corresponding to "critical damping" would cause overshoot.*

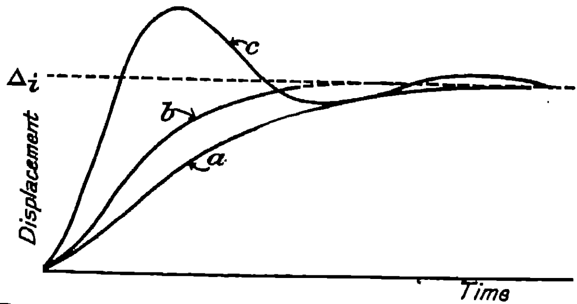


FIG. 7.—DISPLACEMENT-TIME CURVES CORRESPONDING TO VARIOUS DEGREES OF DAMPING

(a) Damping greater than the critical value; creeping deflection
(b) Critical damping; motion dead beat
(c) Damping less than the critical value; overshoot occurs

Fig. 7 shows three curves corresponding to three conditions of the frictional or damping force. Curve 7(*a*) is for a damping force very much greater than critical and the deflection only attains to the value Δ_i, which it should have for the current *i*, after a comparatively long time. Curve 7(*b*) shows the damping force just at its critical value, the deflection attaining its final value in minimum time but without overshooting. Finally, curve 7(*c*) shows the damping force less than critical and the deflection of the wire overshoots its final position by about 37 per cent.

* For parallel electrical case *see* Starling's *Electricity and Magnetism*, pp. 331–341.

In Fig. 7(c) the wire does not come to rest when it returns to the value Δ_i, corresponding to the current i, but oscillates about that value. The rate at which these oscillations die away is so rapid, for any practical value of damping, that it is only the first overshoot that is observable. Thus, if the first overshoot

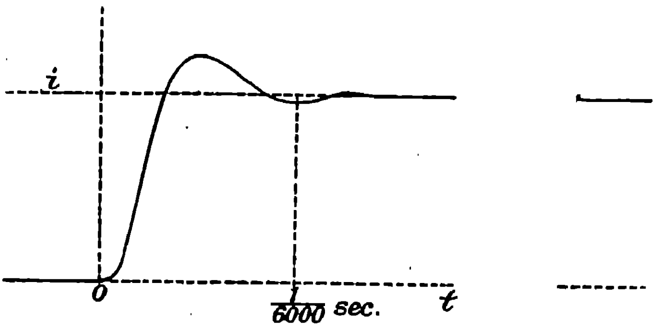


FIG. 8.—SHOWING RAPID CESSATION OF OSCILLATIONS CONSEQUENT UPON OVERSHOOTING

represents 3 per cent of the total deflection, the overshoot on the opposite side of the final position is 3 per cent of 3 per cent, or less than one in a thousand. Even when the first overshoot is as much as 20 per cent, corresponding to about half critical damping, the second overshoot (towards the zero value of current) is only 4 per cent, and the third is less than 1 per cent.

This is shown in Fig. 8 where, in the left-hand diagram, the time base is shown to a very large scale. Thus, if the wire had a natural frequency of 6000 per sec. *when damped*, the complete oscillation occupies a distance of about 2.5 cms. in the left-hand diagram, Fig. 8, and takes $\frac{1}{6000}$ of a sec. In practice the time basis is seldom greater than 400 cm. per sec., and the graphic representation of the curve to this usual scale is shown in the right-hand diagram, Fig. 8. It is seen that the overshoot, instead of being a rounded curve, now appears as a very sharp peak and, in a time equal to $\frac{1}{5000}$ sec., the deflection has practically attained its normal value.

It will be seen from the above considerations that an oscillograph, to give a reasonably true record, must have (*a*) its natural frequency high compared with the frequency it is desired to observe; and (*b*) it must be strongly damped or braked by a frictional force at least equal to the control force at its own natural frequency, when it is called upon to record sudden changes in current.

Motion and Dynamics of a Vibrating Uniform Wire. There are some fundamental relations of a simple

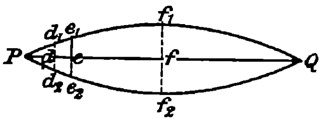


Fig. 9.—Motion in a Vibrating Uniform Wire is Everywhere Perpendicular to the Mean Position

vibrating wire which it is desirable to establish. If a wire be stretched between two points P and Q, Fig. 9, and be made to vibrate between the extreme positions, $Pd_1e_1f_1\,Q$ and $Pd_2e_2f_2\,Q$ then, if there is no longitudinal movement along the wire, the path of movement of every point on the wire will be at right angles to the mean position of the wire $P\,d\,e\,f\,Q$, and a point on the wire, say e, will have a path $e_1\,e\,e_2$.

If the displacement of the wire at the centre $= \Delta$ and if it be assumed for the present that the curve of the

wire when at its maximum displacement is represented by $\Delta \sin x\left(\frac{\pi}{l}\right)$, where x is the distance from P along the wire and l the length of the wire, then the displacement at $(x + dx)$ from $P = \Delta \sin (x + dx) \frac{\pi}{l}$.
The slope of the wire at distance x from P

$$= \Delta\frac{\pi}{l} \cos x\frac{\pi}{l};$$

and the slope of wire at distance $x + dx$

$$= \Delta\frac{\pi}{l} \cos (x + dx,) \frac{\pi}{l}.$$

The angle between the two slopes when the latter are small—

$$= \Delta\frac{\pi}{l} \left\{\cos x \frac{\pi}{l} - \cos (x + dx) \frac{\pi}{l}\right\}$$

$$= \Delta\frac{\pi}{l} \left\{\cos x \frac{\pi}{l} - \cos x\frac{\pi}{l} \cos dx\frac{\pi}{l} + \sin x\frac{\pi}{l} \sin dx \frac{\pi}{l}\right\}$$

$$= \Delta\left(\frac{\pi}{l}\right)^2 dx \sin x \left(\frac{\pi}{l}\right)$$

When dx is so small that $\cos dx\frac{\pi}{l} = 1$ and $\sin dx\frac{\pi}{l}$

$$= dx\frac{\pi}{l}$$

In Fig. 10 there are tangents drawn to the wire corresponding to the ordinates at x and $x + dx$, and shown as T_1 and T_2; the angle between these tangents is that given above.

There is a force acting along these tangents equal to the tension T in the wire, and the resultant of these two tensions is the force accelerating the element of the wire dx to the mean position.

When two equal forces act at a small angle to each other the resultant can be shown to be equal to one of

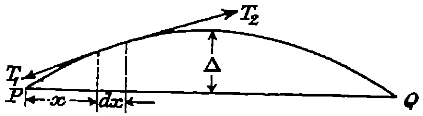


FIG. 10.—ILLUSTRATING THE CONDITIONS IN A SHORT ELEMENT OF A VIBRATING UNIFORM WIRE

the forces times the angle between them. In the above case the force on the element of the wire will be

$$dxT\Delta\left(\frac{\pi}{l}\right)^2 \sin x\frac{\pi}{l} \text{ dynes} \quad . \quad (1)$$

The mass of the wire is m per unit length, hence the mass of length $dx = mdx$.

$$\text{The acceleration} = \frac{dxT\Delta\left(\frac{\pi}{l}\right)^2 \sin x\frac{\pi}{l}}{mdx}$$

and the displacement of the wire $= \Delta \sin x\frac{\pi}{l}$

$$\text{Therefore } f = \frac{1}{2\pi}\sqrt{\frac{T\Delta\left(\frac{\pi}{l}\right)^2 \sin x\frac{\pi}{l}}{m\,\Delta \sin x\frac{\pi}{l}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{T\left(\frac{\pi}{l}\right)^2}{m}} = \frac{1}{2}\sqrt{\frac{T}{lM}} \quad . \quad . \quad (2)$$

Where $ml = M =$ total mass of the wire.

From eqn. (1), the restoring force on unit length of a stretched wire at any portion of its length is

$$T\Delta\left(\frac{\pi}{l}\right)^2 \sin x\frac{\pi}{l},$$

but $\Delta \sin x\frac{\pi}{l}$ is the displacement at that portion, therefore the restoring force per unit length is $T\left(\frac{\pi}{l}\right)^2$ times the displacement.

From eqn. (2), the frequency of vibration of the wire is $\frac{1}{2}\sqrt{\frac{T}{lM}}$ which is the usual expression for a stretched wire.

In the above it has been assumed that the resultant force acts at every point on the wire directly at right angles to the mean position and this is generally very nearly true for any practical amplitude of vibration.

It has also been assumed that the displacement curve is sinusoidal and it is evident that this must be true if there is no control on the wire apart from the tension in the wire itself. Any departure from a sinusoidal curve would mean that portions of the wire would be experiencing a greater or smaller force than would be required to give them the necessary acceleration for the same frequency at every point on the wire. As long as the wire vibrates as a whole and does not give out overtones, and as long as the rigidity of the wire is not appreciable, compared with the force due to the tension, the displacement along the wire is sinusoidal.

Conditions of Vibration in a Loaded Wire. When the wire is loaded in the middle a new condition of affairs exists and, to find the frequency of vibration, it will be assumed that when the wire is loaded it is also shortened as shown in Fig. 11 so that its frequency of vibration remains the same as before.

Let l_1 = the length of the loaded wire; and Δ = the maximum deflection of unloaded wire. If M_2 be the weight applied at the middle of the wire then, as

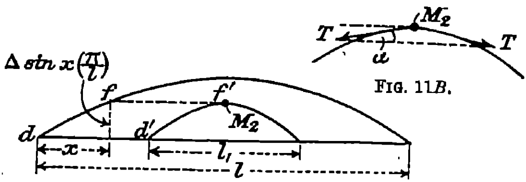


FIG. 11.—UNIFORM AND LOADED WIRES OF EQUAL FREQUENCY OF VIBRATION

the frequency does not vary with the amplitude of vibration, M_2 can be given as maximum displacement equal to $\Delta \sin x\frac{\pi}{l}$; where $x = \frac{l_1}{2}$.

The load M_2 and the point f will then be vibrating with equal amplitude and equal frequency, therefore the portion d^1f^1 of the loaded wire will vibrate in an exactly similar way to the portion df of the unloading wire.

The acceleration of a unit length m, near M_2, will be the same as if it were in the unloaded wire, and will be

$$\frac{T\Delta\left(\frac{\pi}{l}\right)^2 \sin x\frac{\pi}{l}}{m} \quad \text{. . . . . . . (3)}$$

The acceleration of M_2 is $\frac{2T \sin a}{M_2}$, (*see* Fig. 11*B*), but $\sin a = \tan a$, when a is small; and $\tan a$ or the slope of the wire at M_2 is equal to $\Delta \frac{\pi}{l} \cos x \frac{\pi}{l}$, hence—

$$\text{acceleration of } M_2 = \frac{2T\Delta \frac{\pi}{l} \cos x \frac{\pi}{l}}{M_2} \quad . \quad . \quad . \quad (4)$$

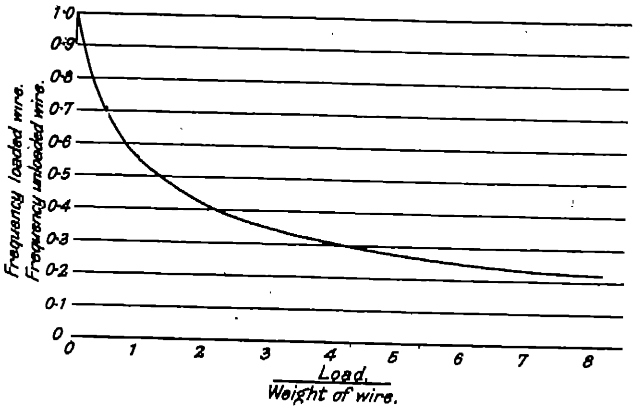


FIG. 12.—EFFECT OF LOADING ON THE FREQUENCY OF VIBRATION OF A WIRE

The acceleration of M_2 and of the small portion near it being equal, we may equate the expressions (3) and (4); i.e.

$$T\Delta\left(\frac{\pi}{l}\right)^2 \frac{\sin x \frac{\pi}{l}}{m} = \frac{2\,T\,\Delta \frac{\pi}{l} \cos x \frac{\pi}{l}}{M_2}$$

$$\therefore \tan x \frac{\pi}{l} = \frac{2}{\pi} \; \frac{M^1}{M_2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where M_1 is the total mass of the longer (and unloaded) wire.

For different values of $\frac{M_1}{M_2}$, x can be found, and $2x =$ the length of the shorter wire. Knowing this, the weight of the short wire can be found and it is the ratio of the added weight M_2 to the *weight of the short wire* that is called the loading.

Against this loading it is only necessary to plot the ratio l_1/l since the frequency of vibration of an unloaded wire is inversely proportional to the length, and such a curve is shown in Fig. 12.

It can be shown that, where the loading is small—

$$\frac{\text{Frequency of loaded wire}}{\text{Frequency of unloaded wire}} = \frac{\text{Weight of wire}}{\text{Weight of wire} + \text{load}}$$

Thus, for a loading weight equal to 20 per cent of the weight of the wire this gives a frequency of vibration equal to 83·3 per cent of the frequency of the unloaded wire, whereas the true value is 84 per cent nearly.

From the two equations (2) and (5) it is possible to obtain by deduction the frequency of any stretched wire when loaded, or of a bifilar system with a mirror placed across it, as long as the torsional forces are small compared with the tensional forces.

CHAPTER II

TYPES OF OSCILLOGRAPHS

Einthoven String Oscillograph (or Galvanometer). This instrument was used originally at lower than what might be called oscillographic frequencies, but with improvements in its construction it has been used for higher and higher frequencies. It consists essentially of a conducting fibre or wire stretched in a strong magnetic field and differs nothing in principle from the simple wire we have been considering in the previous chapter. In practice, the fibre is made very fine, a diameter of 0·02 mm. or less being common when it is made of silver or tungsten and a diameter of 0·002 to 0·003 mm. when it is made of silvered glass.

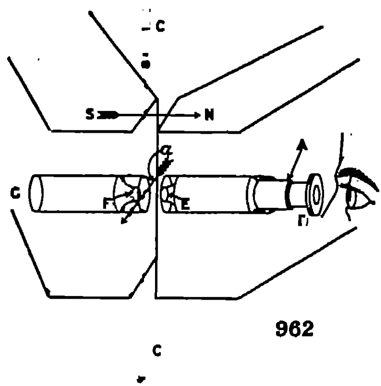


FIG. 13.—DIAGRAMMATIC REPRESENTATION OF THE EINTHOVEN STRING GALVANOMETER

In Fig. 13 is shown a diagrammatic view of a fibre mounted in a strong magnetic field, with one system of lens for strongly illuminating the fibre from some suitable source of illumination, such as an arc lamp, an over-run "gasfilled" lamp, or a Pointolite lamp,

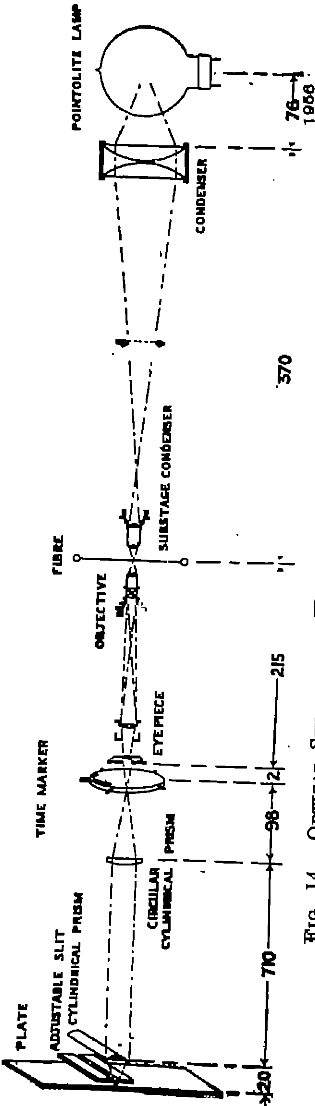


Fig. 14.—Optical System for Use with the Einthoven Galvanometer (Dimensions in millimetres)

and with a second system of lens for observing the movement or projecting the movement on to a screen or photographic plate. Such a lens system for illuminating and observing a fibre of 0·003 mm. diameter is really a microscope, and the optical arrangements normally recommended by the Cambridge Instrument Co. are shown in Fig. 14 where the magnets are omitted for the sake of clearness.

A Pointolite lamp is used as the source of illumination and the light from this is concentrated on the fibre by the main and substage condensers. An image of the illuminated fibre is projected by means of the objective and eyepiece on to the screen or photographic plate.

In the absence of any further lenses this image would appear as a bright circular patch with the fibre showing as a vertical black line across it as shown in Fig. 15(*a*). In order that very rapid movements of the fibre may be observed, this vertical black line must be reduced practically to a point and this is done by two cylindrical lenses with their axes horizontal.

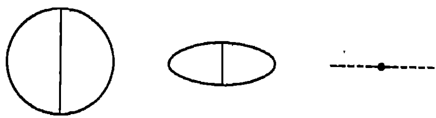


FIG. 15.—ILLUSTRATING THE REDUCTION OF THE IMAGE OF A FIBRE TO A POINT

The first cylindrical lens reduces the circle of light to an eclipse as shown in Fig. 15(*b*), and this is reduced by a second cylindrical lens to a bright illuminated line, shown dotted in Fig. 15(*c*), with the fibre showing as a black spot on this line.

Movements of the fibre result in proportional movements of the black spot along the illuminated line, and the lens system is such that the movement of the fibre is magnified 500 to 600 times on the plate.

With any particular fibre, the sensitivity of the instrument for steady current conditions depends solely on the tension and, as the mass per unit length is very small, the tension need not be high for moderate frequencies.

Thus, for a silver wire 0·02 mm. in diameter to have a frequency of 100 per sec. when its total length is 10 cm. we must have a tension given by—

$$T = 4 f^2 l M.$$

where M is the total mass of the wire

$$= 3{\cdot}3 \times 10^{-4} \text{ grams.}$$

$$T = 4 \times 10^4 \times 10 \times 3{\cdot}3 \times 10^{-4} \text{ dynes.}$$
$$= 132 \text{ dynes} = 0{\cdot}135 \text{ gram.}$$

The force on unit length at the centre of the wire is—

$$\Delta T \frac{\pi^2}{l^2} = \frac{i}{10} B.$$

where Δ = displacement of the wire, in cm.
T = tension, in dynes
l = length of wire, in cm.
i = current, in A
and B = strength of magnetic field, in lines per sq. cm.

Suppose the deflection on the screen is 1 mm. and the magnification is 600, then the actual movement of the wire is $\frac{1}{6000}$ cm. Hence—

$$\frac{1}{6000}\, 132 \frac{\pi^2}{(10)^2} = \frac{i}{10} B$$

and, if B = 20,000 lines per sq. cm., then—

$$i = 1{\cdot}075 \times 10^{-6} \text{ A.}$$

It is seen, therefore, that such an instrument gives roughly 1 mm. deflection for a current of 1 μA when the tension is adjusted to give a natural frequency of 100 per sec.

The damping or frictional force due to the movement of the wire in air is, however, too small to make the instrument anywhere near dead beat and it is only silvered glass fibres of about 0·0025 mm. diameter that are critically damped or dead beat at this frequency.

There are many cases in practice where an instrument of the above sensitivity is very useful, even at commercial frequencies, as the errors introduced owing to the frequency of the measuring oscillograph can be compensated, or the results corrected, as shown in Chapter IV.

To raise the natural frequency of the above instrument to 1000 per sec. would require the tension to be increased to 100 times the former value, i.e. to 13·5 grams, which would be over the safe stress for this size of silver wire.

The length of the fibre can be shortened without loss of sensitivity and without altering the frequency of vibration as, from the equations already given, it can be shown that the current required to give a deflection of Δ cm. is—

$$40\,\pi^2\,\Delta\,m\,f^2\,\frac{1}{B}\,\text{A}.$$

where m is the mass per unit length of the wire.

Therefore the length of the wire does not affect the sensitivity theoretically, as long as the portion of the magnet cut away to accommodate the lenses of the microscope does not form an appreciable portion of the length of the fibre.

In practice, when the fibre is short, this inoperative portion of the fibre becomes very important and lengths shorter than 5 cm. are seldom used. With this length frequencies of 1000 are quite possible, and even frequencies of 2000 per sec. with phosphor bronze wire.

To obtain a photographic record of the movement of the dark image made by the fibre it is necessary to move a photographic plate or film at right angles to the illuminated line along which the dark image of the fibre moves and to exclude from the plate all other appreciable sources of light. A special camera is used to give an even speed to the plate and to cut off stray light.

The last cylindrical lens is generally graduated vertically in millimetres so that these lines appear as faint white horizontal lines on the negative. There is also what is known as a time marker, which periodically cuts off the whole of the light for a very short interval of time. This gives a vertical white line on the negative and, as this cutting off can be arranged to take place at equal intervals, the plate is divided up into equal time intervals and this facilitates precision in the measurements of quantities from the plate.

Fig. 16 is reproduced from such a plate showing records of the voltage given by a normal heart between : (I) The right arm and left arm ; (II) The right arm and left leg ; and (III) The left arm and right leg. The scale of ordinates is 1 mm. = $\frac{1}{10000}$ V, whilst the abscissae are fifths and twenty-fifths of a second for the thick and thin lines respectively.

This reproduction illustrates very well the precision of the instrument for comparatively low frequency measurements. Its range can be extended up to a higher frequency when the electrical method of damping is used.

The record reproduced in Fig. 16 was taken with a silver glass fibre, but very fine silver, aluminium or tungsten wire can be used.

Fig. 17 shows two records taken with an Einthoven galvanometer where the frequency was increased up to 500 per sec. This record was taken on an ordinary cinematograph film running at a speed of 200 cm. per sec. The first portion (*a*) is a record of switching on a current when the damping is that due to air only.

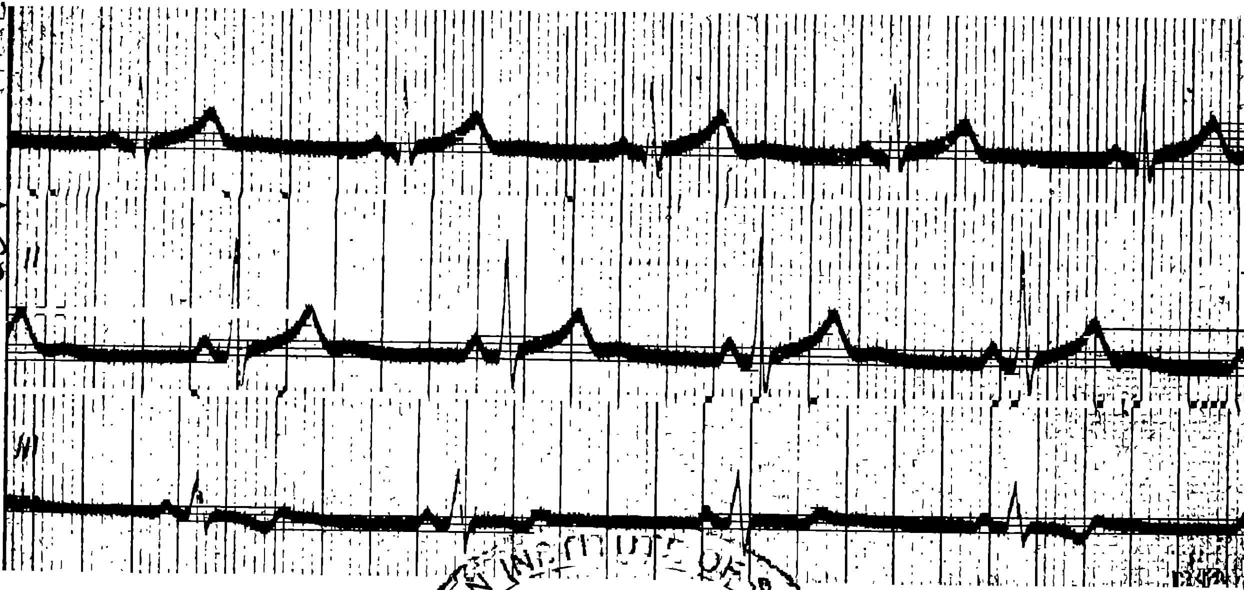
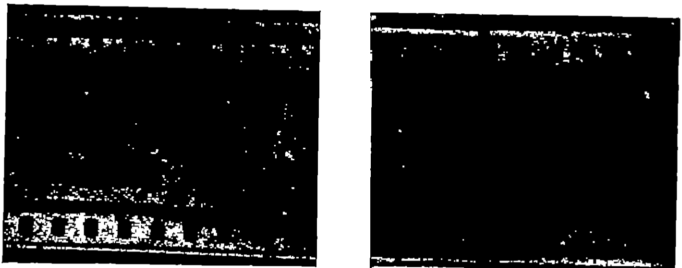


FIG. 16.—TYPICAL RECORD OF HEART ACTION, USING AN EINTHOVEN GALVANOMETER

The second portion (*b*) is a record of switching off a current when the instrument is damped by connecting a resonant shunt in parallel with it as described later. It will be seen that the free vibrations are wiped out in the second case.



(*a*) With air damping alone

(*b*) With damping by a parallel-connected resonant shunt

FIG. 17.—RECORDS OF MAKING AND BREAKING A CIRCUIT USING AN EINTHOVEN GALVANOMETER

Blondel Bifilar Oscillograph. Blondel showed that with a moving coil galvanometer, having a coil suspended by two wires in tension, the sensitivity, at a given frequency, was increased as the number of turns on the coil was reduced, and reached a maximum when the "coil" reached its elemental form of a single loop of wire stretched in a magnetic field with an indicating mirror placed across the loop.

Such a galvanometer or oscillograph is shown in Fig. 18 where a loop of wire *a b c d* is shown stretched across two bridge pieces and placed in the field due to the magnetic *N.S*.

A current passed through the loop, up on one side and down the other, causes one wire to be deflected away from the observer and the other towards the observer.

The relative movement of the wires is indicated by the mirror *M*, stuck across the wires at their middle point and, within limits, the closer the wires are together, the greater the angular deflections of the mirror.

The spot of light reflected from the mirror will move through $4\frac{L}{D}$ times the distance moved by one of the wires; where *L* is the distance of the scale from the mirror, and *D* is the distance apart of the wires.

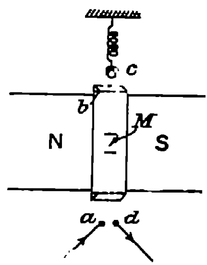


FIG. 18.—DIAGRAMMATIC REPRESENTATION OF BIFILAR OSCILLOGRAPH

In practice, a distance apart of the wires less than 0·30 mm. is rarely possible, and a distance *L* greater than 50 cm. is seldom used when the mirror across the wires is very small. This gives a maximum ratio of movement of spot to movement of wire of 6600—or eleven times the magnification obtained by the Einthoven galvanometer using a magnification of 600—but this advantage is reduced owing to the mirror requiring a much heavier wire to carry it, and owing to the greater tension required by the heavier wire and its mirror load to bring the frequency up to the same value as in the Einthoven instrument.

For very feeble currents and comparatively low frequencies the advantage is altogether with the Einthoven galvanometer, whereas at high frequencies and comparatively large currents the advantage is with the bifilar oscillograph.

Damping. Starting with the bifilar arrangement shown in Fig. 18, the first consideration is how to secure efficient damping.

For low frequencies there is no difficulty, as immersing the wires and mirror in oil of a medium viscosity can be arranged to give all the damping required.

For high frequencies, however, the damping force required is very great and, to secure a sufficient force, Duddell used strips instead of wires and placed them in narrow channels cut in the magnetic circuit with a narrow tongue of iron between the strips.

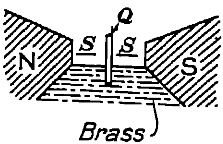


Fig. 19.
Cross-sectional Plan of the Strips and Air-gap in a Duddell Oscillograph

Duddell Bifilar Oscillograph. The general arrangement of this instrument is shown in cross section by Fig. 19, where *Q* is the tongue of soft iron placed between the two strips *S S*. The clearance between the sides of the strips and the channels in which they are placed is so small that when the whole space is filled with oil there is a very considerable force opposing any rapid movement of the wires, and sufficient damping force is obtained. The tongue is cut away at the middle of the strips to allow the mirror to be placed across the strips.

Generally a second element or loop is placed in the same magnetic circuit and close to the first element, so that the two can be illuminated from the same source—preferably an arc lamp.

A complete Duddell oscillograph with two elements, as made by the Cambridge Instrument Co., is shown in Fig. 20, where *C* is an electromagnet for producing a very powerful field between the pole pieces *S S*. The two loops are mounted on independent frames, so that each can be rotated through a

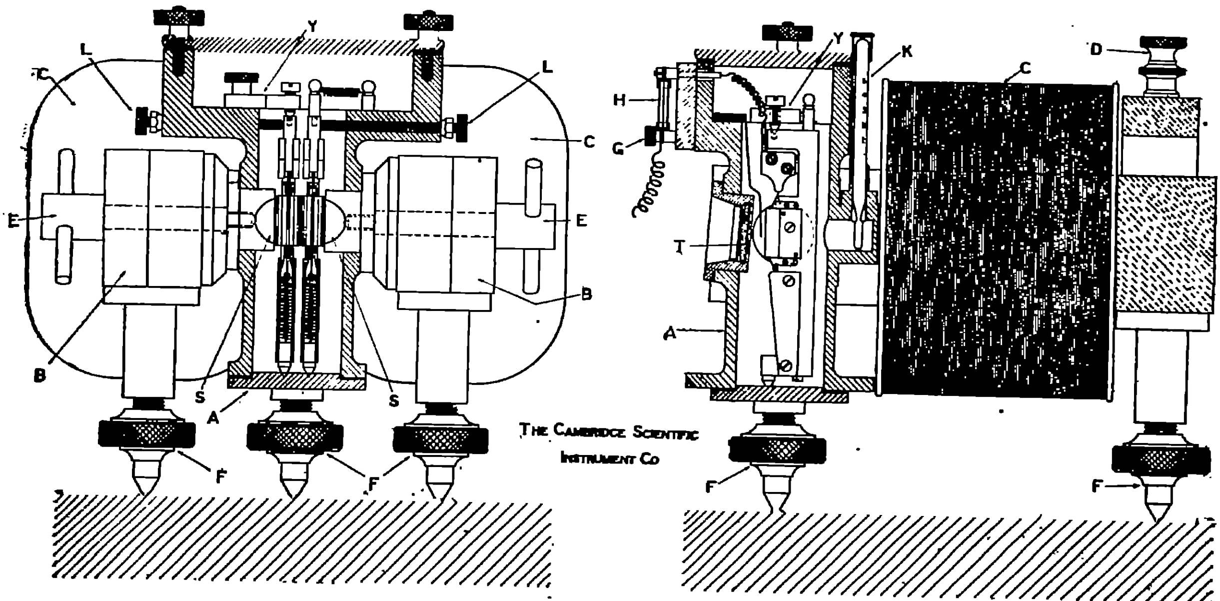


FIG. 20.—SECTIONS THROUGH DUDDELL OSCILLOGRAPH WITH TWO ELEMENTS; SHOWING THE VIBRATORS IN POSITION

small angle by a screw L sufficiently to bring the light spots reflected from the mirrors together on the screen or plate and coincident with the spot reflected from a small mirror placed between the two vibrators.

With the two elements it is possible to obtain two records simultaneously on one plate, e.g. the current flowing in a circuit and the pressure across any part of it, but great care has to be exercised that the difference in electrical potential between the two loops is not large for, owing to the minute clearances between the moving strips and the iron, there is always a danger of a breakdown and consequently disaster to the movements.

The switching arrangements must also be such that the opening of any switch does not allow a difference of potential to exist between the two elements. Generally speaking—and always when fairly high frequency currents are being investigated—the iron case of the magnet should be connected to the common point of the two strips, so as to prevent the flow of current from the edge of the strip to the iron frame.

It is probable that nine-tenths of all the breakdowns of Duddell oscillographs are due to failure to take these precautions.

In the particular model illustrated in Fig. 20, the natural frequency of the vibrating system is about 10,000 per sec. when undamped, that is without oil in the damping chamber, the tension on each loop being 100 grams or 50 grams per strip. The size of mirror is $0{\cdot}3 \times 1{\cdot}0$ mm.

When the damping oil is introduced the frequency of vibration of this instrument in common with all oil-damped oscillographs is reduced, even when the viscosity of the oil is much less than would give efficient damping, and *apart altogether from the lowering of the frequency due to damping.*

The reason for this is that a body in motion in a fluid has energy stored in virtue of its velocity equal to $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} \Sigma m_2 v_2^2$; where m_1 is the mass and v_1 the velocity of the body itself; m_2 is the mass and v_2 the velocity of any small portion of the fluid put in motion by the movement of the body; and $\frac{1}{2} \Sigma m_2 v_2^2$ is the total energy stored in all the fluid by virtue of its motion.

In practice it is found that the effective frequency may be reduced by as much as 33 per cent, corresponding to an increase of effective mass of about 130 per cent, and as the specific gravity of the oil is about one-tenth that of the phosphor bronze strips, the volume of oil put in motion by the strips is some thirteen times the volume of the strips themselves.

This reduction of effective frequency, to 0·66 of the frequency when not immersed, reduces the "factor of merit" of the instrument to 43 per cent of what it is undamped and, in addition it introduces an uncertainty as to what should be called the "natural" frequency of the instrument.

Superimposed on this uncertainty is that due to the change of viscosity of the oil caused by change of temperature; this change of viscosity alters the damping coefficient so that the theoretical correction of the record from an oil-damped instrument is a matter of doubt.

In practice a correction for the magnitude of the deflection of an oscillograph can always be obtained if there is a high frequency alternator available for, if a constant current be kept flowing through the instrument, and the frequency of the current be varied then a curve can be drawn showing the factor at each frequency by which the deflection must be multiplied to make it equal to the deflection at very low frequencies.

The correction for *phase* or time displacement of the deflection from the current producing it is not so easy to determine. If, however, a second oscillograph be available from which the damping oil can be removed then, if the oscillographs have the same current passed through them in series, their records being taken on the same plate, the phase displacement of the damped record can be compared with the undamped at the whole range of frequency of the instruments, and as the phase displacement of the undamped instrument can be easily calculated from its constants, the phase displacement of the record of the damped oscillograph from the actuating current can be calculated.

It may be said, however, that for ordinary commercial work, where harmonics up to an absolute frequency of 1000 per sec. are concerned, no correction is necessary for the above instrument at its normal frequency.

As the size of the mirror in this particular type of instrument is very small it is necessary to illuminate the mirrors by means of an arc lamp, particularly when photographic records are to be taken. The essential arrangement of the optical system as used for this and most other oscillographs is shown in Fig. 21. The light from an arc lamp is focussed so as to illuminate strongly a narrow vertical slit. From this slit the light passes to the oscillograph mirror through a plano-convex lens and from the oscillograph mirror back again through the plano-convex lens to a cylindrical lens, with its axis horizontal, on to a screen or photographic plate.

If it were not for the cylindrical lens an image of the slit would be thrown on the screen by the plano-convex lens in front of the oscillograph, and the slit

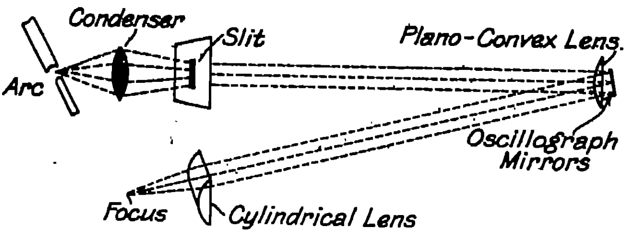
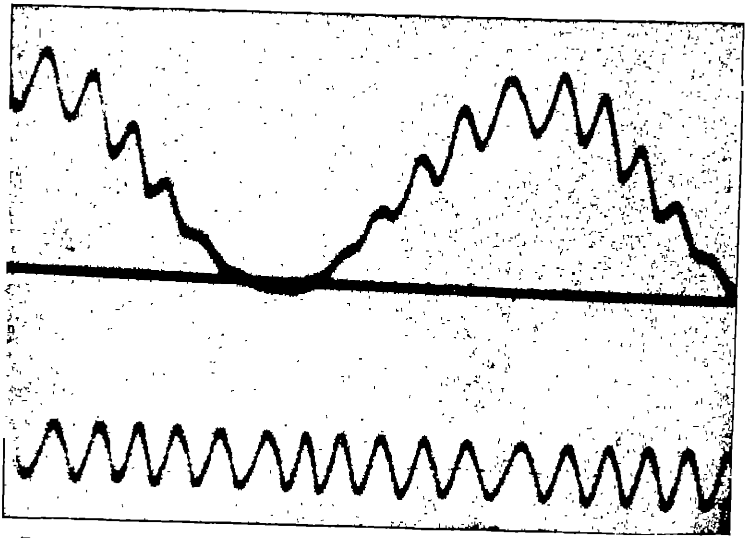


FIG. 21.—DIAGRAMMATIC REPRESENTATION OF THE OPTICAL SYSTEM OF AN OSCILLOGRAPH

and the screen would be conjugate foci for a lens of twice the strength of the plano-convex lens since, in the present case, the light traverses the lens twice. Thus if the screen and the slit were equidistant from the lens then an image of the same size as the slit would be formed. If the slit were $\frac{1}{2}$ mm. wide and 10 mm. high an image $\frac{1}{2}$ mm. wide and 10 mm. high would be given. Such an image would be useless for recording rapid oscillations and it is therefore reduced in height by the cylindrical lens.

If the focal length of the cylindrical lens be, say, 8 cm. and the distance of the lens from the slit *measured along the light path* be 100 cm., then the image of the slit is reduced in height to 0·8 mm. and the spot of light on the screen is nearly a round dot when a certain amount of dispersion is allowed for.

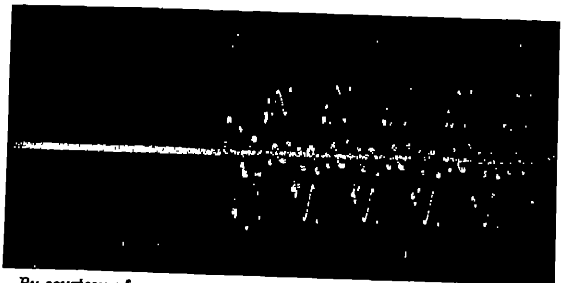
Other Bifilar Instruments. Bifilar oscillographs of the same general type as the above are made by



By courtesy of *Cambridge Instrument Co., Ltd.*

FIG. 22.—SHOWING THE EFFECT OF RESONANCE IN ALTERING THE WAVE FORM OF A ROTARY CONVERTER

Carpentier (Paris), and Siemens & Halske (Berlin), whilst three-element oscillographs are made by the General Electric Co. of America, and by the Westinghouse Electric & Manufacturing Co. The firm last mentioned has developed a type for use in conjunction with an overrun tungsten lamp. This lamp is of the low-voltage, high-current type with a very concentrated filament and it is overrun by about 60 per cent of the normal voltage just at the moment of taking the photographic record on the film. As the lamp is overrun even at what is called the normal voltage, the life of its filament would be extremely short were the 60 per cent excess voltage allowed to persist. It is therefore arranged that the 60 per cent increase is only allowed during the time the shutter of the



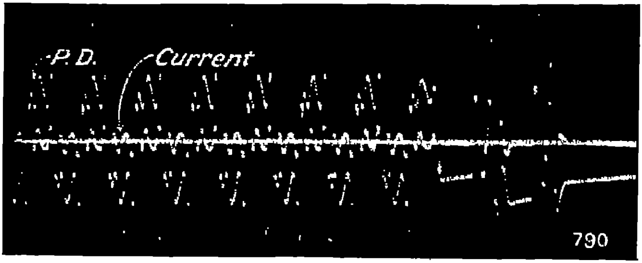
By courtesy of *Cambridge Instrument Co., Ltd.*

FIG. 23.—SHOWING THE RUSH OF CURRENT AND RISE OF VOLTAGE WHEN SWITCHING ON A FEEDER

camera is open, and the switching on of the lamp and the opening of the shutter are arranged to operate from the motor that drives the film, whilst the closing of the shutter opens the switch in the lamp circuit.

These oscillographs also have the three elements insulated from each other by thin micanite sheets, and this is a distinct advantage.

The three reproductions shown in Figs. 22–24 are typical of the records obtained with a Duddell bifilar oscillograph. Fig. 22 shows the effect of resonance in altering the wave form of a rotary converter; Fig. 23 shows the rush of current and rise of voltage at the moment of switching on a feeder; and Fig. 24 shows the fluctuations produced by sparking at the switch contacts at the moment of switching off.



By courtesy of *Cambridge Instrument Co., Ltd.*

FIG. 24.—SHOWING THE FLUCTUATIONS CAUSED BY SPARKING AT THE CONTACTS WHEN SWITCHING OFF A FEEDER

Blondel Moving Iron Oscillograph. This oscillograph, although practically displaced by the bifilar type for general work, has possibilities for special cases, particularly where a small current at a high voltage has to be investigated and where the current sensitivity of the bifilar instrument is not high enough.

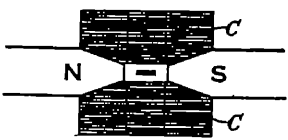


FIG. 25.—DIAGRAMMATIC REPRESENTATION OF THE BLONDEL MOVING IRON OSCILLOGRAPH

The moving portion of the oscillograph is a narrow iron strip placed between the pointed magnets *N* and *S* as shown in Fig. 25. This strip is kept in tension by means of a spiral spring as in the bifilar oscillograph and there is at the same time a very considerable control due to the magnetic field, this acting so as to keep the strip parallel to the lines of force.

There are two coils *CC* placed one on either side of the moving iron strip and with their axles at right angles to the strip and to the lines of force. These coils produce a field at right angles to the main field and cause a slight swing of the main field clockwise or anti-clockwise, depending on the direction of this new field. The iron strip always turns to lie along the resultant field, and as long as the deflection is small the angular movement of the strip is proportional to the strength of the auxiliary field, and therefore to the current in the coils.

These coils can be wound with a few turns to carry a large current or with many turns to carry a very small current. In the latter case the current sensitivity can be made very large. When the coils are wound with many turns their self-induction becomes very large, especially with some of the finer wires now available, but as long as a large enough swamping resistance can be introduced, as on a high voltage system, this is not a serious disadvantage.

The deflection is indicated by a small mirror stuck on to the iron strip and the light passes to the mirror through the hollow front coil. Damping is by means of oil as in the bifilar oscillograph.

In practice it is necessary to use a correction curve obtained by a high-frequency alternator as with the bifilar instrument for the reasons explained on page 36, and also for the following additional reasons—

(1) The effective self induction, the self capacity, and the resistance of the coils vary with the frequency owing to the shunting effect of the capacity from turn to turn and of the capacity from the turns to the iron; this alters the effective impedance of the coil.

(2) The shunting effect of the turn to turn capacity alters the effective ampere-turns on the coil for a given current flowing in the external circuit.

These latter effects can be quite appreciable at the higher frequencies but, where a correction has to be applied, it is not of great importance what particular factor is greatest in causing distortion. The total percentage reduction in the higher harmonics is the important thing to be considered, and if this reduction be large the magnitude of the harmonics in the wave may be so small, compared with the fundamental, that its accurate determination is not possible.

Irwin Hot-wire Oscillograph. Up to the present we have been considering instruments in which the deflection within definite limits is proportional to the current and approximately in phase with it.

The hot-wire oscillograph belongs to a class in which the deflection is not proportional to the current flowing through the instrument, but the latter can be arranged to give a record of the pressure across a circuit or the current flowing in a circuit.

To adapt a hot-wire instrument for use as an oscillograph it is first necessary to make it polarized, so that when the direction of the current reverses, the direction of the deflection is also changed. The method of doing this is shown in Fig. 26, where two fine wires *CD* and *EF* are connected in such a way that a constant direct current from the battery *B* can flow through them in parallel as shown by the arrows *b b*. This current heats both wires equally and if the latter be pulled back by equal tensions at their middle point the mirror *m* placed across the wires will not deflect.

If, however, a current from an external source be sent through the wires in series as shown by the arrows a a, then this current will oppose the current in the left-hand wire and increase the current in the right-hand wire. If the resistance of each wire be r then $\frac{R_1}{R_1 + r}$ of the current flows through each wire. The rate at which heat is given to the left-hand wire is equal to the product of the square of the current multiplied by the resistance $= \left(a \frac{R_1}{R_1 + r} - b\right)^2 r.$ Similarly, the rate at which heat is given to the right-hand wire is—

$$\left(a \frac{R_1}{R_1 + r} + b\right)^2 r.$$

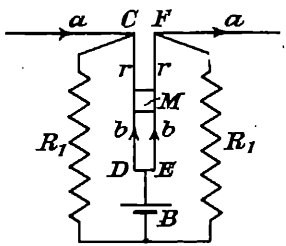


FIG. 26.—ILLUSTRATING METHOD OF POLARIZING IN THE IRWIN HOT-WIRE OSCILLOGRAPH

The difference between these two rates is equal to—

$$-4\,abr \frac{R_1}{R_1 + r};$$

and this expression shows that the difference between the rates of heating is proportional to the external current a, for all the other terms are constant. The difference will be positive or negative according to the direction of current and the direction of deflection of the mirror will therefore change for a change in the direction of the current.

As, however, the wires have a definite amount of thermal capacity they do not reverse their temperature difference immediately the current reverses, and this introduces a time lag between the current and the resulting deflection, but by means of the arrangements now to be described (*see* Figs. 27, 28) it is possible to overcome all difficulties arising from this cause.

Use of Hot-wire Oscillograph to Measure Voltage. Referring to Fig. 27, the polarized wires *CD* and *FE* are connected in series with a condenser *K* across the terminals *AB*; and the condenser *K* is shunted by a

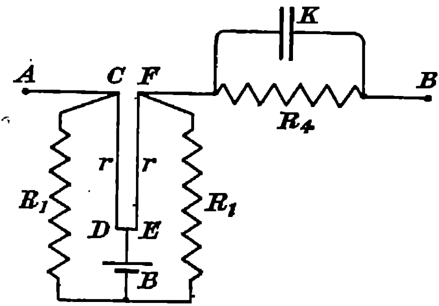


FIG. 27.—ILLUSTRATING ARRANGEMENT OF THE IRWIN HOT-WIRE OSCILLOGRAPH FOR MEASURING VOLTAGE BETWEEN *A* AND *B*

resistance R_4. Then, by suitable adjustment of the constants of the circuit, it is possible to obtain a deflection proportional to the pressure applied between the points *A*, *B*.

Suppose for simplicity that all the current flowing through the condenser and through the resistance R_4 flows through the wires *CD* and *EF* and that the voltage lost in the wires is small compared with that across the condenser and its shunted resistance R_4.

The current flowing into the condenser at any moment is $K\frac{dv}{dt}$ where V is the instantaneous value of the voltage across *AB* (and also across *K* approx.)

The current flowing through the resistance R_4 is $\frac{V}{R_4}$; therefore the total current through the wires is

$$K\frac{dv}{dt}+\frac{V}{R_4}.$$

The difference between the rates of heating of the two wires is, as shown above, $+\ 4\,abr\frac{R_1}{R_1+r}$ and this equals $4\,abr$ very nearly when the term $\frac{R_1}{R_1+r}$ is nearly unity. Substituting for a, the difference between the rates of heating of the wires is given by—

$$4\,br\left\{K\frac{dv}{dt}+\frac{v}{R_1}\right\}$$

When a wire receives heat at a steady rate then its temperature is raised until the loss of heat just balances the gain, but, until this happens, the heat is used up in two ways: part is stored at a rate proportional to $\frac{dT_1}{dt}m\,H_s$; and the rest is wasted by radiation and is equal to $E\,T_1$; where T_1 is the temperature m the mass, H_s the specific heat in joules, and E the emissivity of the wire in joules per sec.

Thus, $\frac{dT_1}{dt}m\,H_s+E\,T_1$ is the rate at which the energy is received by one wire; and $\frac{dT_2}{dt}m\,H_s+E\ \ T_2$ is the rate at which the energy is received by the second wire. The difference between these two rates is the difference between the rates at which the wires receive electrical energy from the current a.

Therefore, $\left\{\frac{d\,(T_1 - T_2)}{dt}\, m\, H_s + E\,(T_1 - T_2)\right\}$

$$= 4\,br\left\{K\frac{dv}{dt} + \frac{V}{R_1}\right\}$$

which may be written for simplicity—

$$k_1\, d\frac{(T_1 - T_2)}{dt} + k_2\,(T_1 - T_2) = k_3\frac{dv}{dt} + k_4 v.$$

Suppose we make $\frac{k_1}{k_2} = \frac{k_3}{k_4}$ then, since the equation holds under all conditions, it will now hold both when $\frac{dv}{dt}$ is large and when it is zero. When $\frac{dv}{dt}$ is zero—

$$T_1 - T_2 = \frac{k_4}{k_2}V \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

and, when $\frac{dv}{dt}$ is very large, so that the terms $k_4 V$ and $k_2\,(T_1 - T_2)$ are negligible—

$$k_1\frac{d\,(T_1 - T_2)}{dt} = k_3\frac{dv}{dt}$$

$$d\frac{(T_1 - T_2)}{dt} = \frac{k_3\,dv}{k_1 dt} = \frac{k_4}{k_2}\frac{dv}{dt} \qquad . \quad . \quad . \quad (2)$$

From eqns. (1) and (2) respectively it will be seen that the current flowing through the shunt resistance R_4 is sufficient to maintain the wires at a difference of temperature equal to $\frac{k_4}{k_2}V$; and, if the voltage changes for any reason at any rate $\frac{dv}{dt}$, the current flowing into the condenser is able to change the difference of temperature at a rate $\frac{d\,(T_1 - T_2)}{dt} = \frac{k_4}{k_2}\frac{dv}{dt}$. This means that the temperature changes are exactly in phase with the voltage and this difference of temperature is always equal to $\frac{k_4}{k_2}V$ so that the difference of temperature of the wires is a measure of the voltage between A and B.

The ratio of k_3 to k_4 is the time constant of the condenser K when shunted by the resistance R_4; and the ratio of k_1 to k_2 is the thermal time constant of the wires. It is therefore necessary to make the time constant of the condenser equal to that of the wires and it is easy to adjust the resistance R_4 so that this is attained.

As long as wires of the same diameter and material are used in the same medium (generally oil) the ratio of K to $\frac{1}{R}$ remains constant. The value of $K R$ for the normal instrument as made by The Cambridge Instrument Co. is 0·007.

Use of Hot-wire Oscillograph to Measure Current. To enable the instrument to indicate the instantaneous current flowing in a circuit it is necessary to have the current flowing in the oscillograph proportional to $M\frac{di}{dt} + Ri$ where i is the current flowing in the circuit and $\frac{M}{R}$ is the thermal time constant of the wires, 0·007 as above. To obtain this value for the current in the oscillograph itself the instrument is shunted across a resistance R in the main circuit and the secondary S of a quadrature transformer, as shown in Fig. 28. The mutual induction between the primary and secondary windings of the quadrature transformer being equal to M, the voltage applied across the oscillograph is $iR + M\frac{di}{dt}$; and, as long as R and M are properly chosen so that the ratio $M/R = 0{\cdot}007$, the deflection of the oscillograph is proportional to the main current within the definite limits of the instrument.

Constructional Features of Hot-wire Oscillograph. In practice it is arranged that the wires $C\,D$ and $F\,E$

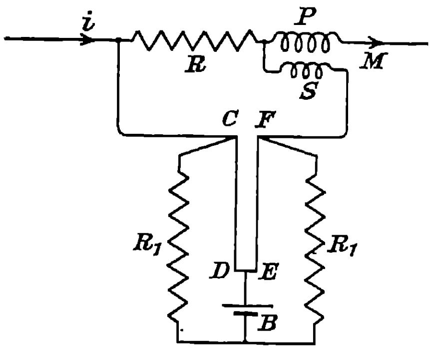


FIG. 28.—ILLUSTRATING ARRANGEMENT OF THE IRWIN HOT-WIRE OSCILLOGRAPH FOR MEASURING CURRENT i IN THE MAIN CIRCUIT

double back on themselves so that each is pulling against the other and only the difference in their movements is observed. The arrangement adopted is shown diagrammatically in Fig. 29 where $C\,D\,C'\,D'$ corresponds to $C\,D$ in Fig. 28, and $E\,F\,E'\,F'$ corresponds to $E\,F$ in Fig. 28. If, in Fig. 29, the wires $C\,D$ and $E\,F$ be looped together at their centre point and kept in steady tension than any difference in the temperature of the two wires will cause a movement of the wires at the centre towards the right or the left. Similarly, the extension of the wires $C'\,D'$ and $E'\,F'$

shown dotted will also produce a movement at their centre due to the same difference in temperature but in an opposite sense, i.e. when the wires nearer the observer move to the right those behind will move to the left. This double movement will cause the small mirror *M* to deflect and give an indication of the movement.

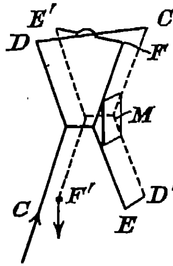


FIG. 29.—ILLUSTRATING METHOD OF TYING TOGETHER THE WIRES OF AN IRWIN HOT-WIRE OSCILLOGRAPH

Sometimes the current is arranged to flow only in the back wires *C D* and *E' F'*, corresponding to *C D* and *E F* in Fig. 28. This is the arrangement used, where the instrument is immersed in oil and where the heated oil rising from the front wires across the mirror *M* would cause a blurring of the spot.

In cases where oil is not used for damping the movement but only for cooling the wires the system shown in Fig. 29 does not carry any current and is not immersed, but is used for magnifying the movement of the current carrying wires as shown in Fig. 30. In this case although the hot-wires CC_1D_1D and EE_1F_1F are extended above the oil and there tied together and provided with a mirror as shown, the portions of the wires out of the oil do not carry any current, as there are cross wires between C_1 and D_1 and also between F_1 and E_1; *see* also Fig. 31. With this arrangement, as the movement is not damped mechanically, there must be some electrical method of damping it, or else controlling the blow given to the wires when the voltage (or current) changes very quickly. To accomplish this, an extra resistance is inserted in the circuit shown in Fig. 27 in series with the oscillograph and in series with the shunted condenser. The effect of this is to slow up any very rapid changes of voltage across the mains, S, and to

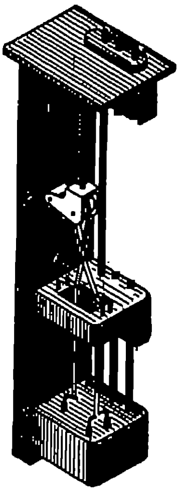


Fig. 30.—System as in Fig. 29, but not Submerged and not Carrying Current, Used to Indicate the Movement of the Hot Wires

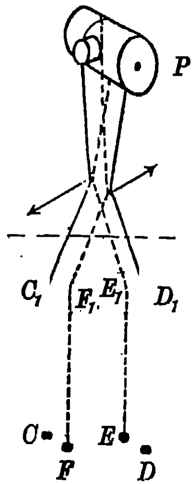


Fig. 31.—Enlarged Sketch of the Wires in Fig. 30

decrease the maximum current flowing through the oscillograph into the condenser.

Consider this first from the point of view of change of frequency. If there be a constant potential between the points A and B, and if the resistance of the oscillograph be very small, the current flowing into the condenser K will be directly proportional to the frequency. If, however, the oscillograph itself and the added resistance be in series with the condenser the current is no longer proportional to the frequency but, as the frequency is raised, the current tends to a finite value equal to the voltage divided by the sum of the two resistances, and the actual value is easily found by means of vectors.

The ratio of the current so found to the current that would flow if there were no series resistance for the condenser is a measure of the reduction of deflection due to damping. There is, however, an increase of deflection (up to a limit) for an undamped instrument owing to the control force being opposed by the inertia forces (*see* Fig. 4). The product of these two factors gives the real magnification (or attenuation) for the particular damping used.

In the arrangement (Fig. 28) used when investigating the current flowing in a circuit, the ratio secondary ampere-turns to the primary ampere-turns on the quadrature transformer gives a measure of the damping on the element, it can easily be increased by connecting a resistance in parallel with the secondary circuit and across the oscillograph itself—that is, if the resistance of the oscillograph is not low enough to produce sufficient damping.

Abraham Rheograph. If a moving coil galvanometer were used as an oscillograph there would be very serious errors introduced owing to the inertia of the coil and the damping if this were appreciable, but Abraham has succeeded in making these effects negligible by introducing new forces to balance them.

On p. 5 it is shown that, for a simple wire placed in a magnetic field, there are only four possible forces and the sum of these at every instant is equal to zero thus—

$$\frac{i}{10} B - k \Delta - \rho v - am = 0$$

but $$v = \frac{d \Delta}{dt} = \text{and } a = \frac{d^2 \Delta}{dt^2}$$

Therefore $\frac{i}{10} B - k \Delta - \frac{d\Delta}{dt} \rho - \frac{d^2 \Delta}{dt^2} m = 0$

It is obvious that it is only when $\frac{d \Delta}{dt}$ and $\frac{d^2 \Delta}{dt^2}$ are negligible that $\frac{iB}{10} = k \Delta$ and $i = \frac{10}{B} \Delta k$.

In order that i may always be equal to $\frac{10}{B} \Delta k$ we have to send two auxiliary currents through the wire; the value of the first of these is $\frac{di}{dt} \frac{\rho}{k}$ and the value of the second is $\frac{d^2 i}{dt^2} \frac{m}{k}$

The current through the wire would then be—

$$i + \frac{di}{dt} \frac{\rho}{k} + \frac{d^2 i}{dt^2} \frac{m}{k}$$

but since $\frac{iB}{10}$ is a force on the wire it can be expressed in terms of the control force by $k \Delta_1$ where Δ_1 is the deflection required to give a force $\frac{iB}{10}$ under steady conditions. Then the total current

$$i + \frac{di}{dt} \frac{\rho}{k} + \frac{d^2 i}{dt^2} \frac{m}{k}$$

will produce an electro-magnetic force—

$$k\,\Delta_1 + \frac{\rho}{k}\frac{d\,(k\,\Delta_1)}{dt} + \frac{m}{k}\frac{d^2\,(k\,\Delta_1)}{dt_2}$$

$$= k\Delta_1 + \frac{d\Delta_1}{dt}\,\rho + \frac{d^2\,\Delta_1}{dt^2}\,m$$

but this force plus $\left\{ -k\,\Delta - \frac{d\Delta}{dt}\,\rho - \frac{d^2\Delta}{dt^2}\,m \right\} = 0$

since the sum of the electro-magnetic and mechanical

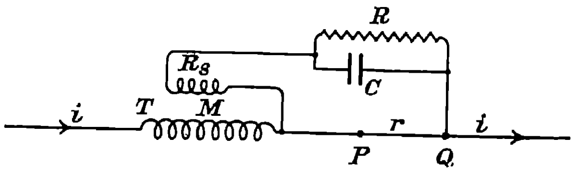


FIG. 32.—ILLUSTRATING ONE METHOD OF COMPENSATING FOR INERTIA AND DAMPING IN THE ABRAHAM RHEOGRAPH

forces must be zero, therefore Δ_1 must always be equal to Δ and $\frac{iB}{10} = k\,\Delta$; that is, no matter what the current is at any moment, or how rapidly it is changing, the deflection will be proportional to it as long as we are able to arrange that the current through the rheograph is equal to $i + \frac{di}{dt}\frac{\rho}{k} + \frac{d^2i}{dt^2}\frac{m}{k}$

In Fig. 32 one way (although not the original arrangement given by Abraham) of arranging a circuit to accomplish this is shown, where i is the current in the main circuit that has to be examined. In this diagram PQ is, say, a wire stretched in a magnetic field and having a resistance r; C is a condenser and R a resistance in parallel with it; and T is a transformer which must fulfil the condition that the flux is proportional at every instant to the primary current, so that the voltage induced on the secondary winding is equal to $M\frac{di}{dt}$ where M is the mutual induction between the windings.

To fulfil this condition the current in the secondary must be so small that the ampere-turns on the secondary winding are small compared with those on the primary; and either there must be no iron in the magnetic circuit or, if there is, there must be a large air gap in the path. A practical value for the secondary ampere-turns would be about $\frac{1}{10}$ the primary ampere-turns at the highest frequency it was desired to record, say, 1000 per sec. corresponding to the 20th harmonic in a wave of 50 frequency. If great accuracy were required the ratio of secondary to primary ampere-turns would have to be made still smaller.

If a steady current i flow in the main circuit, part of this current will flow through the wire PQ and part through the secondary of the transformer and the resistance R.

The current in the wire is $\frac{R + R_s}{R + R_s + r} i$ and this can be made practically equal to i.

There is a voltage induced on the secondary of the transformer equal to $M\frac{di}{dt}$ and a current flows from this winding through R and the wire PQ. The value of this current is $M\frac{di}{dt} / (R + R_s + r)$ and is very nearly equal to $\frac{M}{R}\frac{di}{dt}$, when R is large compared with R_s and r.

There is also a current flowing from the secondary of the transformer $= \frac{d\left(M\frac{di}{dt}\right)}{dt}C$; and, as long as the impedance of the condenser is large compared with R_s and r,

$$\frac{d\left(M\frac{di}{dt}\right)}{dt}C = M\,C\frac{d^2i}{dt^2}$$

Therefore the conditions that have to be established are that $\frac{M}{R} = \frac{\rho}{k}$ and that $M\,C = \frac{m}{k}$ but it is not necessary to know ρ, m, and k, the constants of the instrument as long as we know its frequency and the ratio of the actual damping to that required to give critical damping. It is shown on pp. 11 and 12 that, at the resonant frequency of the instrument, the control and the inertia forces are equal and opposite. Also, it is shown above that in the expression

$$i + \frac{di}{dt}\frac{\rho}{k} + \frac{d^2i}{dt^2}\frac{m}{k}$$

for the actual current through the wire PQ, the first term balances the control force, the second the damping force, and the last the inertia force. Therefore—

$$\frac{d^2i}{dt^2}\frac{m}{k} = M\,C\frac{d^2i}{dt^2} = i$$

at every moment when the instrument is working at its own resonant frequency; or, writing *R.M.S.* values—

$$\omega^2 M C A = A$$

where A is the current in the main circuit then

$$M C = \frac{1}{\omega^2}$$

If an instrument be critically damped then, at its own resonant frequency, the force due to the damping is twice the control force. Suppose the actual damping in an instrument is $\frac{1}{n}$th of that required to give critical damping, then the maximum force due to damping is $\frac{2}{n}$th of k, the maximum control force, and the current required to give this force is $\frac{2}{n}$th of the current A in the mains. Thus, $\frac{2A}{n}$ = current from the secondary through the resistance R; but this secondary current

$$= \frac{\omega M A}{R + R_s + r} \text{ or nearly } = \frac{\omega M A}{R}$$

Therefore $\quad \frac{2A}{n} = \frac{\omega M A}{R}$ and $\frac{M}{R} = \frac{2}{\omega n}$.

To work out an example of applying the above reasoning to a concrete case, take the Einthoven string galvanometer mentioned on p. 25 where the silver fibre is 0·02 mm. diameter, has a resistance of about 6 ohms, a natural frequency of 100 per sec., and a damping (say) $\frac{1}{10}$th of critical damping.

If M be made small, C has to be large and R small, so it is a matter of compromise what value to make M. Suppose M be taken as 0·1 henry, then—

$$M\,C = \frac{1}{\omega^2}$$

$$C = \frac{10}{(2\,\pi\,100)^2} = 25{\cdot}2 \times 10^{-6} \text{ farads,}$$

$$\text{and } R = \frac{M\,\omega\,n}{2} = \frac{0{\cdot}1 \times 6280}{2} = 314 \text{ ohms.}$$

An Einthoven galvanometer made into an Abraham rheograph by the mutual induction and the condenser would retain its current sensitivity *at all frequencies and would be without distortion if* R_s *and* r *were made small compared with the impedance of the condenser.*

In the present case the impedance of the condenser at 1000 frequency would be about 6 ohms and, if the resistance R_s were, say 6 ohms also, the current flowing into the condenser would only be $\dfrac{6}{\sqrt{6^2 + 12^2}}$ of what it ought to be to give no change of sensitivity and no distortion—that is, the higher harmonics of frequency about 1000 would be 45 per cent of their true value.

In practice, care has to be taken that the resonance point in the instrument does not come into the working range for, although there should be no error introduced at the resonance point if the adjustments be made correctly, yet at that point we have two currents of large values which are equal and opposite and two forces which are equal and opposite, hence any variation of the natural frequency of the instrument would upset the balance point and give a deflection much greater than the true value.

This is a reason for having fairly heavy damping as when the instrument is so damped no such irregularity occurs as at the resonance frequency if the natural frictional force be large. Care has also to be exercised to avoid electrical resonance but the danger of this is small, if the mechanical damping be large, for R is made less as the mechanical damping increases and this low resistance across the condenser and the resistance of the wire r damp out the electrical oscillations.

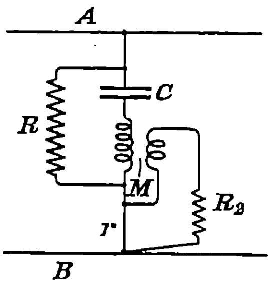


FIG. 33.—ILLUSTRATING THE USE OF A RHEOGRAPH TO RECORD THE INSTANTANEOUS PRESSURE ACROSS A B

Fig. 33 shows an arrangement of a rheograph to record the instantaneous value of the pressure across a circuit A B. In this diagram, r represents the resistance of the instrument itself; R, a large series resistance across the mains giving a term $\frac{V}{R}$ to balance the control force; and C a condenser which, apart from the primary of the mutual induction, is also practically across the mains giving a term $C\frac{dv}{dt}$ to balance the damping or frictional force. The primary of the inductance has a current $C\frac{dv}{dt}$ and, if M be the mutual induction, the current in the secondary is $C\,M\,\frac{d^2v}{dt^2} / (R_2 + r)$; which is the term that balances the inertia force of the instrument.

As the inertia forces can be so easily balanced Abraham has made rheographs carry mirrors of 25 sq. mm. area or, say, 80 times the area of those on the Duddell high frequency oscillograph ; and, in the first instance, the method was applied to a moving coil galvanometer which shows its possibilities.

Nevertheless, every increase in the inertia force means corresponding increase in the size of the condenser and the mutual inductance and a corresponding reduction of the sensitivity, and is therefore to be deprecated.

Electrostatic Oscillographs. Oscillographs, in which the deflecting force is the pull of an electrostatic field, have been used (unpolarized) by Professor Gray, and polarized by Einthoven, by the author, and by Drs. Ho and Kato of Tokio University.

Some of the difficulties and successes encountered during a research extending (although not continuous) over some seven years are summarized below, partly to show the convenience of the instrument for some special problems, and partly to show the difficulties to be encountered and avoided.

All the mechanical problems present in the electromagnetic oscillograph are also present in the electrostatic instrument, and it is necessary to have the mechanical frequency high and the damping large if the instrument is to indicate correctly.

As, however, the maximum electrostatic force present in air before breakdown occurs is small, it is generally necessary to immerse the moving parts in oil for three reasons : (1) To get a greater electromotive force between the plates, owing to the greater dielectric strength of oil as compared with air ; (2) To get a stronger field owing to the specific inductive capacity being higher; (3) To obtain sufficient damping.

On the other hand, the introduction of oil into the field, in addition to increasing the inertia of the moving part, has the disadvantage that, if it is not perfectly homogeneous, the oil will separate under the strong electrostatic field, the oil of higher dielectric coefficient and greater conductivity being drawn between the plates into the strongest part of the field.

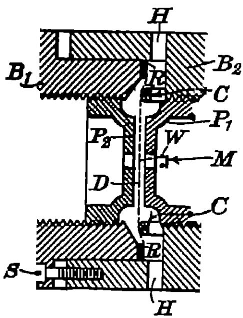


FIG. 34.—SECTIONAL PLAN OF THE IRWIN ELECTROSTATIC OSCILLOGRAPH

This field also attracts any particles of dust or foreign matter and great care has to be taken to select a pure oil, to filter into the containing vessel through dry blotting paper and to keep it clean. Any accidental spark between the plates in the oil often upsets the whole experiment, and the oil may have to be filtered again or fresh oil used.

Of the polarized instruments, the type used by the author was that in which a diaphragm made of very fine gauze, about $(90)^2$ meshes per sq. cm., was placed between two inductor plates and the general arrangement of the instrument is shown in Fig. 34, which is a horizontal section through the instrument.

Two blocks of ebonite B_1 and B_2 are turned to fit into one another as shown. The gauze diaphragm D is soldered on to a brass ring R, which fits inside the block B_2 and is pushed in front of the block B_1. When the diaphragm comes up against the annular ring $C\ C$, which is part of the block B_2, it is stopped and any further movement of B_1 simply stretches the diaphragm and alters its natural frequency. The movement of B_1 relative to B_2 is made by three screws S placed at 120 degrees round the block B_1.

The inductor plates, P_1 and P_2, are screwed into the blocks B_1 and B_2, and the movement of the diaphragm D is rendered visible by attaching a piece of silk to it and bringing this through the hole in the centre of P_1 to one of two cross wires W which are placed vertically. It is arranged that this cross wire stands slightly in front of the parallel wire so that when it is pulled back by the silk it has a sag which always keeps the silk taut so that any small movement of the diaphragm, back or forward, is communicated to the wire. If a small mirror be placed across the wires W it will have a tilting movement proportional to the movement of the diaphragm.

There are a number of holes H all round the block B_2 to facilitate the entrance of oil at the bottom of the instrument and the escape of air at the top during the filling of the instrument, as the presence of even minute air bubbles between the plates would lead to local ionization or even breakdown especially where the gap between the plates and the diaphragm is small, in some cases about 0·25 mm.

The whole instrument is immersed in a bath of oil in which there is a suitable window for allowing of the illumination of the mirror very much as shown in Fig. 20 for the Duddell oscillograph.

The connections of an electrostatic oscillograph to indicate pressure are shown in Fig. 35, where a battery of total voltage $2P$ is shown connected across the two plates of the oscillograph and the source of voltage V to be observed is connected between the middle point of the battery O and the diaphragm.

The voltage between the upper plate and the diaphragm is $P + V$ if the instantaneous voltage given by the source be as shown. The pull on the diaphragm is proportional to the square of the applied voltage, and is, therefore, equal to $K_1 (P + V)^2$. Similarly, the voltage between the lower plate and the diaphragm is $(P - V)$ and the pull on the diaphragm is $K_2 (P - V)^2$. Now K_1 and K_2 can be made equal by adjusting the distance of the plates, and the resultant pull on the diaphragm is then equal to $K\{(P + v)^2 - (P - v)^2\} = 4KPV$ and, as K and P are both constant, the deflection of the diaphragm (and of the mirror) is proportional to V, the instantaneous voltage applied, and will reverse when the voltage reverses. This assumes, however, that the natural frequency of the diaphragm and of the wires is sufficiently high to enable them to follow the voltage changes, and that the diaphragm and wires are sufficiently well damped to prevent them having appreciable free vibrations of their own. A frequency (in oil) of about 1500 per sec. is attainable.

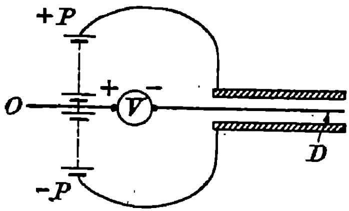


FIG. 35.—CONNECTION DIAGRAM FOR AN ELECTROSTATIC OSCILLOGRAPH USED TO INDICATE PRESSURE

The essential calculations for a moving diaphragm are comparatively simple as long as the diaphragm is assumed to be moving everywhere with simple harmonic motion and as long as every part of the diaphragm moves in the same phase, an assumption probably true for a light tightly stretched diaphragm.

If the required movement of the spot of light on the screen or photographic plate is 2 cm., and if the magnification is 6600 say, then the movement of the diaphragm is $\frac{2}{6600}$ or 3×10^{-4} cm.

If the resonant frequency of the instrument be 1500 in oil and 2300 per sec. say, in air then, when the diaphragm is vibrating at its own frequency over a distance 3×10^{-4} cm. on each side of the mean position, the restoring force must be always equal to the inertia force if there is no damping present.

The maximum inertia force $= \omega^2 \Delta m$; where Δ is 3×10^{-4} cm., and m is the mass of the diaphragm in grm. per sq. cm. As the wires in the gauze in the present case are 0·0034 cm. diameter, m is 0·015 grm; and the restoring force per unit area at the centre of the disphragm is—

$$\begin{aligned} \omega^2 \Delta m &= (2\pi\, 2300)^2 \times 3 \times 10^{-4} \times 0{\cdot}015 \\ &= 940 \text{ dynes.} \end{aligned}$$

It is immaterial to what this restoring force is due—whether to tension in the wires or stiffness; the essential is that it must have this value for the given displacement.

Now if the displacement under an electrostatic pressure is to have the same value stated, the force produced by the electrostatic field must be equal to 940 dynes per sq. cm.

The electrostatic pull between two parallel plates is equal to—

$$F = \frac{V^2 k}{8\pi t^2}$$

where V = difference of potential in electrostatic units; k = the specific inductive capacity of the dielectric = 2, say; and t = the distance apart of the plates = 0·025 cm. in the case considered.

Therefore $$V^2 = \frac{940 \times 8\pi\,(0{\cdot}025)^2}{2}$$

V = 2·7 electrostatic units or 2·7 × 300 Volts = 810 Volts.

In the above examples it is assumed that there is only a pull on one side of the diaphragm and, to achieve this, it is desirable that the value of P, which is half the voltage of the polarizing battery, should be roughly equal to the maximum voltage from the source to be investigated: that is that $(P - V)$ should be small compared with $(P + V)$.

The reason for the use of a gauze diaphragm, instead of a thin sheet metal diaphragm, is that if the oil were not permitted to move through the diaphragm it would be impossible to get rapid movements owing to the mass of oil that would have to be put in motion.

The example is worked out above for a "gap" of 0·025 cm. This is about the smallest practical and generally a gap of 0·5 cm. is better when, with a polarizing voltage $P = 2000$, a sensitivity of about 2 cm. for a voltage of 1300 maximum, or, say, an alternating sinusoidal pressure of 900 V (*R.M.S.*), would give a total swing of 4 cm.

Einthoven Electrometer or Electrostatic Oscillograph. To find the force on electrostatic oscillographs in which there is a wire carrying a charge and placed in an electric field it is necessary first of all to know the capacity of the wire per unit length.

If a wire of radius r be placed at a distance d from an earthed plate and have a positive charge q per unit length, then, by the theory of images, if a negative charge q per unit length be placed on a similar wire at an equal distance d on the opposite side of the plate, the plate could be removed without disturbing the lines of force.

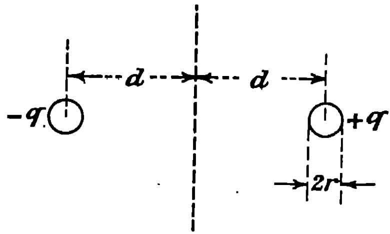


FIG. 36.—ILLUSTRATING THE CALCULATION OF THE CAPACITY OF A WIRE WITH REGARD TO A PLATE

The work done on a unit charge in moving it from the surface of one conductor to the surface of the other conductor against the field strength of *one charge*, is

$$\int_{x=(2d-r)}^{x=r} \frac{2q}{x}\,dx = 2q \log_e \frac{2d-r}{r}$$

The work done against both fields is twice this $= 4q \log_e \frac{2d-r}{r} =$ the pressure between the conductors.

The capacity is equal to the charge divided by the pressure hence the capacity of one wire with regard to the other $= \dfrac{q}{4q \log_e \frac{2d-r}{r}} = \dfrac{1}{4 \log_e \frac{2d-r}{r}}$ but, since the work done in moving the charge over half the distance, i.e. from the position of the plate up to the charged conductor, is only $\frac{1}{2}$ V, the capacity of the original conductor with regard to the plate is twice the capacity from wire to wire and is equal to

$$\frac{1}{2\log_e \frac{2d-r}{r}}$$

Since r is small compared with d the expression for the capacity of the wire with regard to the plate may be taken to be $\frac{1}{2\log_e \frac{2d}{r}}$.

If the wire be placed midway between two parallel plates the capacity of the wire with regard to the two plates is twice the above or $\frac{1}{\log_e \frac{2d}{r}}$ but $2d$ is the distance of the plates apart $= D$, so that the capacity of a wire of radius r, midway between two parallel plates which are a distance D apart, is $\frac{1}{\log_e \frac{D}{r}}$. If the wire be immersed in oil of specific inductive capacity k the capacity $= \frac{k}{\log_e \frac{D}{r}}$ electrostatic units.

Now the force on the wire per unit length will be equal to the field strength times the charge per unit length divided by the specific inductive capacity.

The field strength E is equal to the pressure V_1 between the two plates times the specific inductive capacity divided by the distance D between the plates or $E = V_1k/D$

Therefore the force on the wire per unit length =

$$\frac{V_1 k}{D} \times \frac{k}{\log_e \frac{D}{r}} \times \frac{V_2}{k}$$

where V_2 is the pressure of the wire above the mean of the pressures of the two plates. This expression reduces to—

$$\text{Force on wire, per unit length} = \frac{V_1\, V_2\, k}{D \log_e \frac{D}{r}} \text{ dynes,}$$

where V_1 and V_2 are measured in electrostatic units.

Now the maximum force permissible depends on the maximum charge we can safely give to the wire and the maximum field we can allow between the plates.

The maximum stress due to the charge on the wire is at the surface of the wire, and for a voltage V_2 is equal to $\dfrac{2\,V_2}{r \log_e \frac{D}{r}}$

Since this stress is superimposed on that due to the main field the maximum stress due to the charge on the wire can be taken as half the allowable stress in the medium. For oil a working stress of 3000 V per millimetre is allowable ; hence the permissible stress due to the charge on the wire is 1500 V per mm., or 50 electrostatic volts per cm. Therefore—

$$\frac{2V_2}{r \log_e \frac{D}{r}} = 50$$

To apply this to a specific example, let us take the case corresponding to that worked out for a silver wire 0·002 cm. diameter carrying a current in a magnetic field, (p. 25) and suppose that in the present case the wire is carrying a charge in the electrostatic field due to two parallel plates 1 mm. apart.

Then $D = 0{\cdot}1$ cm. and $r = 0{\cdot}001$ cm.; and, from these values, the maximum value of $V_2 = 0{\cdot}5$ electrostatic units approximately (or 150 V). If the maximum stress due to the voltage between the parallel plates be also 50 electrostatic volts per cm.—

V_1 maximum = 5 electrostatic units (or 1500 V) and the maximum force per unit length

$$= \frac{5 \times 0{\cdot}5 \times 2}{0{\cdot}1 \times 2{\cdot}3 \log_{10}{}^{100}} = 11 \text{ dynes}$$

which would be equal to the force developed by a current of 0·01 A flowing in a wire placed in a field of 11,000 lines per sq. cm.

Such a wire we found (on p. 26) required a force of about $1 \times 10^{-5} \times 20{,}000$ dynes to give a deflection of 1 mm. when used with a microscope of magnification 600; so that an effective force of 0·2 dyne gave a deflection of 1 mm. If, therefore, in the present case, the wire were again placed under such tension that its frequency was 100 per sec. the deflection for a polarizing voltage of 1500 (max) or $P = 750$ V and 150 V on the wire would be 55 mm.

As ionization does not generally take place, even with the wire in air, at less than 300 V, V_2 may be increased to 300 when the value given by the above reasoning is below 300. In the present instance, with $V_2 = 300$, the deflection would be 110 mm. If the least workable deflection be 20 mm., the frequency of the wire can be increased to 100 $\sqrt{5 \cdot 5}$ or 230 per sec.; but how much higher the deflection can be pushed or the frequency increased depends on how near one works to the breakdown point.

An electrostatic oscillograph with a single wire placed between two parallel plates is used very much as shown in Fig. 35 except that the diaphragm D (shown in cross section in Fig. 35) is replaced by a single wire. The recording apparatus as shown in Fig. 14 is used. As the sensitivity does not vary with the length of the wire, so long as the frequency is constant, it is advisable in the electrostatic instrument—especially for the higher frequencies—to make the length of the wire short, for this adds to the stability of the fibre. As the beam of light passes at right angles to the field there is no inactive part of the field as in the corresponding electromagnetic instrument.

Ho and Kato Electrostatic Oscillograph. This instrument has a pair of wires placed in an electrostatic field and so arranged that when the potential of the one increases above zero the potential of the other falls below zero by an equal amount. The arrangement is shown diagrammatically in Fig. 37 which is a horizontal section through the instrument. P_1 and P_2 are the two field plates; two wires W_1 and W_2, shown in section; are placed close together and carry a mirror M which is illuminated by a beam of light through the opening O. The wires are insulated from each other and from the mirror.

The force on each wire can be calculated from the equations given for the Einthoven instrument, and for two wires sufficiently close together the magnification may be of the order of 6000. The deflection, with certain sizes of wire, may therefore be ten times as great as for the corresponding Einthoven instrument. For very small wires, where the mass of the mirror is appreciable compared with mass of the wires, the advantage is reduced. For smaller sizes still, the use of a mirror is inadmissible and the advantage in point of sensitivity is then with the Einthoven instrument, especially where a silver glass fibre is used, and even though the smaller Einthoven wires have to be used in air.

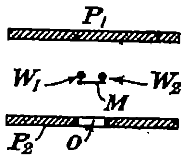


FIG. 37. DIAGRAMMATIC REPRESENTATION OF THE HO AND KATO ELECTROSTATIC OSCILLOGRAPH

One method of connecting the Ho and Kato instrument is shown in Fig. 38, where $P_1\,Q_1$ are the terminals connected to the plates, and $P_2\,Q_2$ the terminals

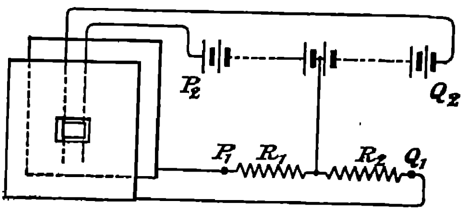


FIG. 38.—CONNECTIONS FOR THE HO AND KATO ELECTROSTATIC OSCILLOGRAPH

connected to the two wires. The potential between P_1 and Q_1 can be divided by two equal resistances $R_1\,R_2$ connected in series. If a polarizing battery be connected between P_2 and Q_2 the middle point of this battery can be connected to the junction of R_1 and R_2. The voltage to be observed is now connected to $P_1\,Q_1$.

The reverse connections can also be used where R_1 and R_2 are connected between P_2 Q_2 and the polarizing battery between P_1 Q_1.

Scope of Electrostatic Oscillographs: It will be seen from the foregoing that electrostatic oscillographs are suitable for comparatively high voltages and for wave forms where a very high frequency in the indicating instrument is not required.

The great advantage of this type of instrument, however, is that the current required to operate it is extremely small and does not upset the working conditions. The current required by the instrument itself is only the charging current required to raise its potential and, as the capacity is small (ranging from 40×10^{-6} μF in the stretched diaphragm to ·5 $\times 10^{-6}$ μF for the Einthoven instrument considered) the current required is very small, even at frequencies as high as 1000 per sec. The current taken by the instrument itself may, in fact, be much smaller than that taken to charge the wires which connect it to the apparatus under test.

Another great advantage of the electrostatic instrument is the ease with which it can be applied to circuits of very high voltage. It is only necessary to connect a suitable condenser in series with the instrument and the voltage is then divided up between the condenser and the oscillograph in the inverse ratio of their capacities. If oil be used in the oscillograph, the same sort of oil must be used in the condenser, so that the electrical time constants of the oscillograph and the condenser are equal.

Power Measurements by Electrostatic Oscillograph. An electrostatic oscillograph can be used to show the instantaneous power given to a circuit. Fig. 39 shows the well-known arrangement of an electrostatic wattmeter to measure mean watts, but, if the instrument be of high enough frequency, it will show the instantaneous watts or the power being given to the circuit A B no matter what the type of circuit may be. This is exactly true if the moving wire (or diaphragm) be connected to the middle point of the resistance R_1, but the instrument measures the power in the circuit A B plus half the power being wasted in R_2 if the moving wire be connected to the point B directly or through a condenser. A new use can be found for an electrostatic oscillograph wattmeter to measure power at frequencies much higher than the instrument can follow. Thus, in wireless telegraphy, such an instrument can show the amount of power given to the aerial at, say, 100,000 cycles per sec. when the aerial is excited by means of a "spark" method.

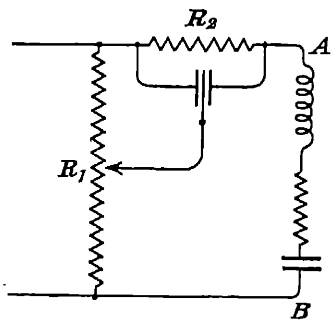


FIG. 39.—ILLUSTRATING THE PRINCIPLE OF THE ELECTROSTATIC WATTMETER

Under such conditions, where the damping of the oscillations is small, the rate at which power is given to the aerial can be shown by a curve which might have a shape as shown at Fig. 40(*a*). In this diagram, the heavy line gives the mean of the impulses due to the rapidly varying power in the aerial which the instrument is unable to follow. If the damping be large, or the frequency high, then the instrument will only have a throw given to it as shown in Fig. 40(*b*). In that case the instrument will have to be calibrated as a *ballistic wattmeter* by discharging a condenser through a resistance when the power given to the resistance is known to be $\frac{1}{2} KV^2$ if the circuit can be closed quickly enough.

In using electrostatic instruments where the mechanical damping is not sufficient it is desirable to use a series

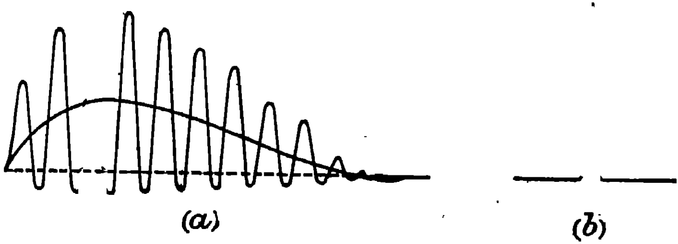


FIG. 40.—ILLUSTRATING THE MEASUREMENT OF HIGH FREQUENCY POWER

resistance to give some electrical damping, especially as this constitutes a safeguard at the same time.

As the instrument is a condenser the maximum rate at which it can be charged is equal to V/R, where R is the series resistance and V is the applied voltage across the resistance and oscillograph.

If Q_o be the charge that would eventually be stored in the oscillograph if the applied voltage remained constant, and if Q be the charge after a time t, then it can be shown that

$$t = CR \log_e \frac{Q_o}{Q_o - Q} \text{ sec.}$$

where C is the capacity of the oscillograph and R is the series resistance.

If $Q_o - Q$, the portion of the charge which has still to flow in, be expressed as a fraction, $\frac{1}{n}$th of the final charge, then—

$$t = 2{\cdot}3\ CR \log_{10} n \text{ sec.}$$

When $n = 10$, i.e. when there is a tenth of the charge still to flow in—

$$t = 2{\cdot}3\ CR \text{ sec.}$$

so that nine-tenths of the charge and of the voltage of a condenser has been attained in a number of seconds equal to 2·3 times the time constant.

Suppose, for example, that the oscillograph has a capacity 40×10^{-12} farads and $R = 10^6$ ohms ; then, for $\frac{1}{n} = \frac{1}{10}$:—

$$t = 2{\cdot}3 \times 40 \times 10^{-12} \times 10^6$$
$$= 0{\cdot}92 \times 10^{-4} \text{ sec.}$$

that is, in a time equal to $\frac{1}{10000}$ sec., the voltage will have already attained 90 per cent of its final value, and in $\frac{1}{5000}$ sec. it will attain 99 per cent of its final value.

It is obvious, therefore, that a megohm in series with the instrument will not produce appreciable distortion of the curve, and even 5 megohms will produce less distortion than normal oil damping.

The resistance of 5 megohms is put in series for a pressure of say, 1000 V, so the current that would pass if the oscillograph broke down is only $1000/(5 \times 10^6)$ A or 200 μA. This would also be the maximum current that would flow through the experimenter if he accidentally touched the oscillograph, provided that the *only* incoming source of high potential was through the resistance of 5 megohms, and that this high resistance was able to stand up to the high voltage across it.

If the same oscillograph were used on, say, 40,000 V the series condenser would be one-fortieth of 40×10^{-12} farad or 1×10^{-12}; and the series resistance would be 200 megohms, which would again only allow the same small current as in the preceding case. A diagram of the connections for such an electrostatic oscillograph working on 40,000 V is given in Fig. 41. In practice a small portion, say one megohm, of the series resistance is put in series with each side of the polarizing battery, so as to reduce the current from the battery in case of a breakdown of the oil in the oscillograph due to some excess of applied pressure.

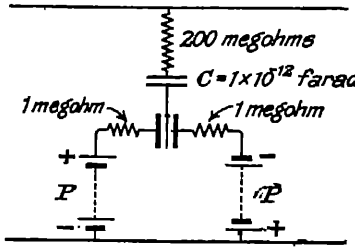


Fig. 41.—Connections for an Electrostatic Oscillograph Working on 40,000 Volts

Figs. 42–46 are reproduced from records taken by the author during an investigation into the variation of wave on the secondary of a high tension testing transformer of 4 kW output.

This transformer had a transformation ratio of 140 to 40,000 V and, owing to its comparatively small size and high voltage, resonance could take place between the leakage self-induction and the self-capacity of the secondary. When the alternator speed was adjusted so that resonance took place corresponding to

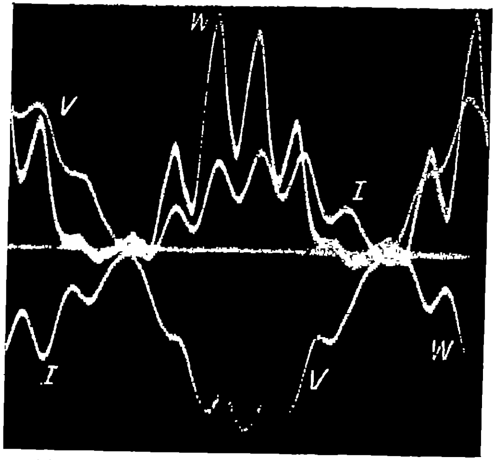


FIG. 44.—RECORDS SHOWING THE POWER SUPPLIED FROM THE SECONDARY OF A HIGH TENSION TRANSFORMER TO A WATER LOAD

(*Note.*—The voltage curve is reversed for clearness)

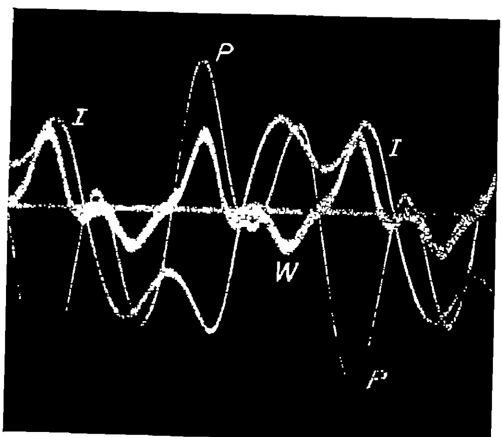


FIG. 45.—RECORDS SHOWING POWER SUPPLIED FROM THE SECONDARY OF ONE TRANSFORMER TO A SIMILAR TRANSFORMER PLACED BACK TO BACK WITH IT

(*Note.*—The voltage curve is reversed for clearness)

Figs. 44–46 are records illustrating the use of the electrostatic oscillograph as a wattmeter. The curve marked *W* represents the instantaneous power, and *P* is the pressure curve, and *I* is the current curve for the primary. Fig. 44 is for the power supplied from the secondary of the high tension testing transformer

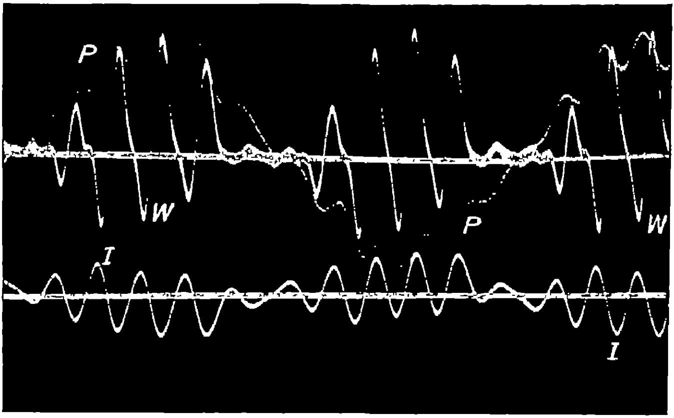


FIG. 46.—RECORDS SIMILAR TO THOSE IN FIG. 45, BUT WITH THE ALTERNATOR USED FOR FIGS. 42 and 43
(*Note.*—Voltage curve reversed and current curve displaced for clearness)

mentioned above, to a water resistance, the primary current and pressure being measured on a hot-wire oscillograph. Fig. 45 is a record of the power given by the secondary of the transformer to a similar transformer placed back to back with it, the current and pressure waves on the primary again being measured by a hot-wire oscillograph. The records in Fig. 46 were taken under conditions similar to those for Fig. 45, except that the current wave was displaced for the sake of clearness and the alternator was the same as for Fig. 42. The wave *W* in Fig. 46 shows very clearly the rapid surging of power between the two transformers as well as the slower flow of power to supply the losses in the second transformer.

Cathode Ray Oscillographs. The cathode ray oscillograph involves principles which have not been discussed up to this point; a complete chapter has, therefore, been devoted to a description of its working and its uses (*see* p. 126).

CHAPTER III

ERRORS OF INDICATION—METHODS OF DAMPING—AND NEW METHODS OF CONNECTION

As an oscillograph cannot always be of sufficiently high frequency, compared with the frequency of the source being investigated, it is sometimes necessary to correct the curves of the instrument where the highest accuracy is required. The relation of the control, damping, and inertia forces is shown in Fig. 4 (p. 10) and the fact is there established that the resultant of these sources must balance the force due to the current in the instrument.

Magnification with and without Damping. If the damping of the instrument be very slight corresponding to, say, an Einthoven oscillograph with a fairly heavy wire (say, 0·02 mm. diameter), or to a Duddell oscillograph without damping oil, then the only effective force is the resultant of the control and the inertia forces.

The maximum inertia force for a sinusoidally varying deflection is $\omega^2 \Delta m$ and, for a constant swing, is therefore proportional to the square of the frequency.

At the resonance frequency, f_R, of the oscillograph the maximum inertia force is equal to the maximum control force, hence—

$$f_R{}^2(2\pi)^2 \Delta m = \Delta K$$
$$\text{or } f_R{}^2(2\pi)^2 m = K.$$

At any frequency f at which the oscillograph is operated—

$$f^2(2\pi)^2 m = (f/f_R)^2 K$$

but the resultant internal force is the difference between the control and the inertia forces, and equals $K - (f/f_R)^2K$ at a frequency f when the damping term $\rho\omega$ is negligible.

This is shown in Fig. 47 for unit deflection and where the vectors K and $\omega^2 m$ or $(f/f_o)^2K$ are drawn at an angle π to each other. The resultant control for unit deflection varies therefore from K at zero frequency to $K - (f/f_R)^2K$ at frequency f. The deflection for unit current in the oscillograph is inversely proportional to the control, therefore—

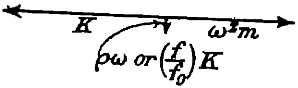


FIG. 47.—VECTOR DIAGRAM OF THE FORCES IN AN OSCILLOGRAPH WITH VERY LITTLE DAMPING

$$\frac{\text{Deflection at frequency } f}{\text{Deflection at zero frequency}} = \frac{K}{K - (f/f_R)^2K} = \Omega \text{ (say)}$$

This ratio Ω is the magnification and equals

$$\frac{1}{1 - (f/f_R)^2}.$$

This magnification also applies to an oscillograph of any type when it is not subjected to any appreciable damping. In Fig. 48 is shown the magnification Ω plotted against the ratio f/f_R. At the resonance point, when $f = f_R$, the magnification, in the absence of damping, would be infinity. In practice it may be 200.

When the instrument is damped the vector $\rho\omega$, which has been neglected so far, represents an appreciable force and the resultant mechanical force for the oscillograph is required in terms of the three vectors K, $\rho\omega$ and $\omega^2 m$.

If, taking the general case, the damping be n times critical damping then, since $\rho_c\omega = 2K = 2\omega^2 m$, we have: $\rho\omega = n\rho_c\omega = 2nK = 2n\omega^2 m$ at the resonant

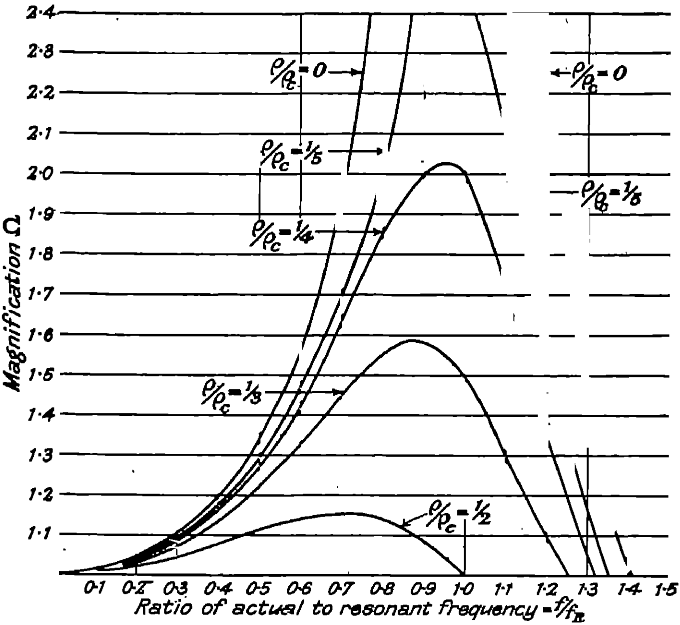


FIG. 48.—CURVES SHOWING MAGNIFICATION AS A FUNCTION OF THE RATIO (WORKING FREQUENCY/RESONANCE FREQUENCY), FOR DIFFERENT VALUES OF DAMPING

frequency. Let the ratio of the frequency at which the oscillograph is working to the resonant frequency $(= f/f_R) = k$, then—

$$\rho\omega \text{ at frequency } f = 2nKk$$

$$\text{and } \omega^2 m \text{ at frequency } f = (f/f_R)^2 K = k^2 K.$$

Then the resultant mechanical force, obtained by compounding the three vectors is—

$$1\sqrt{(2nKk)^2 + (K - k^2K)^2}$$
$$= K\sqrt{k^4 + k^2(4n^2 - 2) + 1}$$

but, when $\omega = 0$, the force $= K$; therefore the force at frequency f is $\sqrt{k^4 + k^2(4n^2 - 2) + 1}$ times as large as at $f = 0$, and the deflection for a constant current is—

$$\frac{1}{\sqrt{k^4 + k^2(4n^2 - 2) + 1}}$$

times deflection at frequency $= 0$. If $n = \frac{1}{20}$, corresponding to a lightly damped instrument, then—

$$\frac{1}{\sqrt{k^4 + k^2(4n^2 - 2) + 1}} = \frac{1}{\sqrt{k^4 - 1{\cdot}99k^2 + 1}}$$

If a second curve were plotted on Fig. 48 to represent this amount of damping, it would lie so close to the curve for the undamped oscillograph as to be indistinguishable from it over the range of magnification covered by this figure. It is therefore apparent that, over a range of frequency from 0 to 75 per cent of the resonant frequency, an oscillograph without damping oil gives a magnification for all practical purposes as if it had absolutely no damping.

In Fig. 48 curves are also drawn for the magnification with damping equal to one-fifth, one-fourth, one-third, and one-half of the critical damping, and these curves resemble the usual resonance curves for partially damped resonance circuits.

Lag of Deflection behind Applied Force. Fig. 49 shows the angle by which the deflection lags behind the applied force for different values of damping from $\rho/\rho_c = \frac{1}{40}$ to $\rho/\rho_c = \frac{1}{2}$; and for varying frequencies relative to the resonant frequency of the instrument.

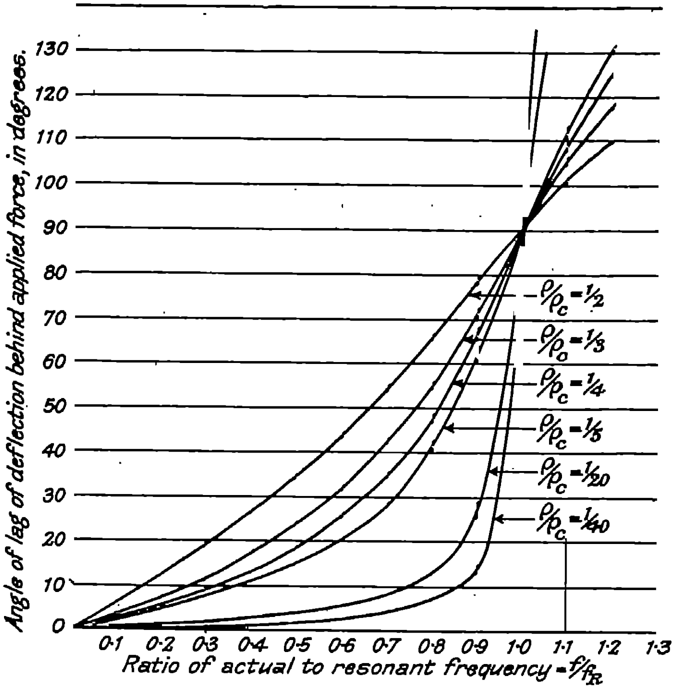


FIG. 49.—CURVES SHOWING LAG OF DEFLECTION BEHIND APPLIED FORCE AS A FUNCTION OF THE RATIO (WORKING FREQUENCY/RESONANCE FREQUENCY), FOR DIFFERENT VALUES OF DAMPING

The value $\rho/\rho_c = \frac{1}{40}$ corresponds to the value found for a certain high-frequency Duddell oscillograph when there was no oil in the damping chamber. To find the angle of lag it is necessary to compound the three vectors K, $\omega^2 m$, and $\rho\omega$ or [(since $\omega^2 m = K(f/f_R)^2$ and $\rho\omega = 2K(\rho f)/(\rho_c f_R)$] the three vectors K, $K(f/f_R)^2$ and $2K(\rho f)/(\rho_c f_R)$.

Call $\frac{f}{f_R} = k$ and $\frac{\rho}{\rho_c} = n$ as before; then the vectors to be combined are K, Kk^2, and $2Knk$; and are as

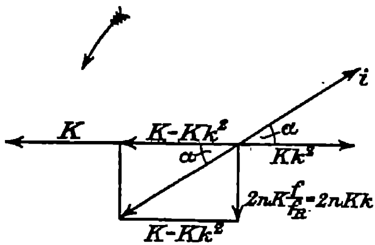


Fig. 50.—Vector Diagram Showing Combination of Forces to Determine Angle of Lag of Deflection

shown in Fig. 50. From this diagram it will be seen that—

$$\tan\alpha = \frac{2nKk}{K - Kk^2} = \frac{2nk}{1 - k^2}.$$

When n is small $\tan\alpha$ and α are also small. If n were zero, there would be no angle of lag for any frequency up to the resonant frequency, and when the frequency was increased above resonance the angle α would swing over from zero to a lag of π.

If $\rho = \frac{\rho_c}{40}$ (to take a possible case where a complex wave form contained a frequency, 75 per cent of the resonant frequency and a frequency one-twentieth of this again, i.e. 3·75 per cent of the resonant frequency) then at 75 per cent of the resonant frequency $\alpha = 5°$ ·

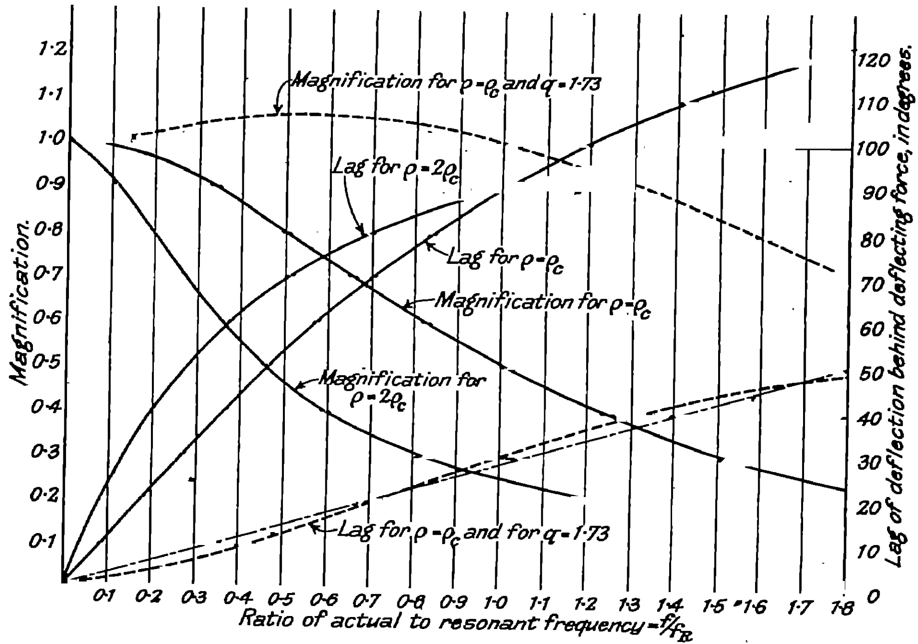


FIG. 51.—CURVES SHOWING MAGNIFICATION AND LAG OF OSCILLOGRAPH DEFLECTION AT VARIOUS FREQUENCIES, WITH CRITICAL AND TWICE CRITICAL DAMPING; ALSO FOR THE SPECIAL CASE $\rho = \rho_c$ AND $q = 1{\cdot}73$, CONSIDERED LATER (P. 116)

and at 3·75 per cent of the resonant frequency the displacement $a = 0{\cdot}1°$ (approx.). The relative displacement of the two vibrations *to the scale of the lower frequency* $= 0{\cdot}1° - {\cdot}25° = -0{\cdot}15°$ which would be quite impossible to detect on a curve.

It is therefore seen that an undamped oscillograph can be used for recording *periodic waves* as long as the highest frequency to be recorded does not exceed 75 per cent of the resonant frequency of the instrument. Such an instrument will give negligible phase distortion of the harmonics and will exaggerate the magnitude of harmonics. The latter characteristic is often an advantage in that it makes it easier to detect the harmonics, and as the exaggeration *is very definite and a correction can easily be applied*, it is not a disadvantage where the curve has to be analysed.

In Fig. 51 curves are given showing the magnification of an oscillograph for critical damping and for twice critical damping plotted against the ratio of the frequency to the resonant frequency of the oscillograph. There is also plotted on the same sheet the phase difference (lag in this case) between the deflection and the deflecting force.

A comparison can now be made from Figs. 48, 49, and 51, between the (practically) undamped oscillograph ($\frac{1}{40}$th of critical) and the oscillograph critically damped. The results of such a comparison are tabulated in Table I.

It is seen that, even if the oscillograph could be damped without adding to the inertia of the moving parts, the undamped oscillograph gives a more nearly accurate record of the wave at the lower frequencies. Thus, at a frequency 0·2 of the resonant frequency, the undamped instrument makes the wave 4 per cent too large and the damped instrument makes it 4 per cent too small. The phase displacement is 0·57° in the undamped instruments, and 22·5° with critical damping. Actually, if the undamped instrument be immersed in oil to secure damping, its resonant frequency is reduced owing to the inertia of the oil put in motion. Any comparison between the two cases, for a given frequency of applied current, should, therefore, be made for a ratio f/f_R for the undamped and for a ratio $1{\cdot}5f/f_R$ for the damped instrument.

If a ratio $f/f_R = 0{\cdot}2$ be chosen for the undamped instrument then $f/f_R = 0{\cdot}3$ should be chosen for the instrument with damping.

TABLE I

Comparison Between Undamped and Critically Damped Oscillographs

Ratio f/fR	Undamped Oscillograph		Critically Damped Oscillograph	
	Magnification	Phase Displacement in degrees	Magnification	Phase Displacement in degrees
0·1	1·01	0·28	0·99	11·4
0·2	1·04	0·57	0·96	22·5
0·3	1·1	0·9	0·92	30·4
0·4	1·19	1·4	0·85	44·0
0·5	1·33	1·9	0·8	53·0
0·6	1·56	2·7	0·735	62·0
0·7	1·96	3·9	0·67	69·5
0·8	2·77	6·35	0·61	77·0
0·9	5·26	13·5	0·555	83·5
1·0	—	90·0	0·5	90·0
1·1	4·76	165	0·453	95·5
1·2	2·27	172	0·41	95·5

On this basis, the undamped instrument makes the record (at $f/f_R = 0{\cdot}2$) 4 per cent too large and the critically damped instrument makes it 8 per cent too small, the relative phase displacement being 0·57° and 30·4° in the respective cases. When one is dealing *with periodic waves only* it is better to use an undamped instrument as long as the highest frequency to be recorded is less than 70 per cent of the resonant frequency of the instrument.

The undamped instrument could not be used, however, in a circuit where there were any sudden variations of current or potential as these would cause free vibrations of the instrument and it might be difficult to separate the free vibrations from those due to the current variations to be recorded.

Desiderata in Regard to Damping. From the foregoing it is evident that it would be an advantage to be able to damp the movement of the oscillograph without adding to the inertia of the moving parts. It would also be a great advantage if the amount of the damping could be varied at will and yet be perfectly definite at each value.

With this in view the author began some experiments in 1907 to find out the necessary conditions for damping an electro-magnetic oscillograph by means of a suitable shunt.

It is known that the damping of moving coil galvanometers is very much increased by short circuiting the coil through a low resistance but the damping produced by the movement of the coil in the magnetic field is altogether inadequate for damping even a high frequency galvanometer, let alone an oscillograph. For example, consider a galvanometer with a natural period of

1 sec. and just dead beat when short circuited on itself. If the period be shortened to $\frac{1}{100}$ sec., the maximum control force will be 10,000 times as great as before, but the maximum damping force will be only 100 times as great as before, so that the electro-magnetic

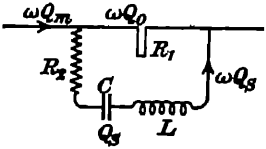


FIG. 52.—CONNECTIONS FOR THE IRWIN RESONANT SHUNT FOR ELECTRO-MAGNETIC OSCILLOGRAPHS

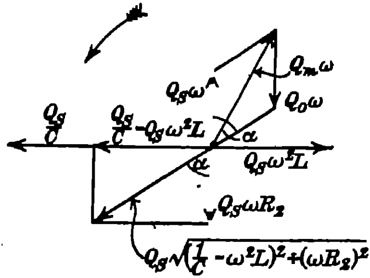


FIG. 53.—VECTOR DIAGRAM FOR THE CONDITIONS SHOWN IN FIG. 52

damping will only be $\frac{1}{100}$th of that required to secure dead-beat conditions.

It is evident, therefore, that putting a shunt across an electro-magnetic oscillograph will not damp out free vibrations of the wires or stripes and in one respect this is fortunate, for otherwise the damping of an oscillograph would vary with the value of the shunt used when recording current values in a circuit.

Irwin Method of Damping Electro-magnetic Oscillographs. In Fig. 52 is shown the arrangement of the Irwin resonant shunt where the oscillograph of resistance R_1 is shown shunted by a circuit containing a condenser of capacity C, an inductance L, and a resistance R_2. If a *periodic* current i_m in the mains be written ωQ_m, where Q_m is the maximum displacement in either direction round the circuit, then Q_s is the maximum charge in the condenser C, and the maximum current in the shunt circuit is $Q_s\omega$. Q_o is the maximum quantity, and $Q_o\omega$ is the maximum current through the oscillograph.

The vector representing these conditions is shown in Fig. 53, where the current in the shunt, $Q_s\omega$ is drawn vertically and the back voltage due to the resistance R_2 is shown at an angle π with $Q_s\omega$. The back voltage Q_s/C of the condenser is shown leading on the current $Q_s\omega$ by $\frac{\pi}{2}$ and the voltage $Q_s\omega^2L$ lagging by $\frac{\pi}{2}$.

The resultant back voltage is

$$Q_s\sqrt{\left(\frac{1}{C} - \omega^2L\right)^2 + (\omega R_2)^2}$$

and lags by an angle a behind $Q_s\omega R_2$, but this resultant voltage is also the voltage given by $\omega Q_o R_1$, therefore, $Q_o\omega$ and

$$Q_s\sqrt{\left(\frac{1}{C} - \omega^2L\right)^2 + (\omega R_2)^2}$$

are at an angle π to each other and the angle between $Q_o\omega$ and $Q_s\omega$ is a where cos a

$$= \frac{\omega R_2}{\sqrt{\left(\frac{1}{C} - \omega^2L\right)^2 + (\omega R_2)^2}}$$

If the voltage across the oscillograph and shunt $= V$

then $Q_o = \dfrac{V}{R_1\omega}$ and $Q_s = \dfrac{V}{\sqrt{\left(\frac{1}{C} - \omega^2L\right)^2 + (\omega R_2)^2}}$

$$Q_m = \sqrt{Q_o^2 + Q_s^2 + 2Q_oQ_s\cos a}$$

$$= V\sqrt{\left(\frac{1}{R_1\omega}\right)^2 + \frac{1}{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2} + \frac{2\omega R_2}{R_1\omega\sqrt{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2}\sqrt{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2}}}$$

$$= V\sqrt{\left(\frac{1}{R_1\omega}\right)^2 + \frac{1}{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2} + \frac{2R_2}{R_1\left\{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2\right\}}}$$

$$= V\sqrt{\frac{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2+(R_1\omega)^2+2\,R_2R_1\omega^2}{\left\{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2\right\}(R_1\omega)^2}}$$

$$= V\sqrt{\frac{\left(\frac{1}{C}-\omega^2L\right)^2+\omega^2(R_1+R_2)^2}{\left\{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2\right\}(R_1\omega)^2}}$$

$$\frac{Q_o}{Q_m} = \frac{V}{VR_1\omega}\sqrt{\frac{\left\{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2\right\}(R_1\omega)^2}{\left(\frac{1}{C}-\omega^2L\right)^2+\omega^2(R_1+R_2)^2}}$$

$$= \sqrt{\frac{\left(\frac{1}{C}-\omega^2L\right)^2+(\omega R_2)^2}{\left(\frac{1}{C}-\omega^2L\right)^2+\omega^2(R_1+R_2)^2}}$$

The ratio $\frac{Q_o}{Q_m}$ is the reduction in the deflection that takes place due to the presence of the resonant shunt, but the current ωQ_o through the oscillograph produces (as shown on p. 84) a magnification

$$\frac{1}{\sqrt{k^4 + k^2(4n^2 - 2) + 1}}$$

times as great as it would produce at a frequency very near zero. *Therefore, the ratio of the deflection of the oscillograph with shunt at frequency* $\frac{\omega}{2\pi}$ *to the deflection of oscillograph without shunt at a very low frequency—*

$$= \sqrt{\frac{\left(\frac{1}{C} - \omega^2 L\right)^2 + (\omega R_2)^2}{\left\{\left(\frac{1}{C} - \omega^2 L\right)^2 + \omega^2(R_1 + R_2)^2\right\}\left\{k^4 + k^2(4n^2 - 2) + 1\right\}}}$$

and this is the general expression for an oscillograph so shunted and with a damping n times as great as critical. Generally n is only a small fraction.

To change from the general case to the particular, where the resonant frequency of the electrical circuit used as a shunt is equal to the resonant frequency of the oscillograph, we have—

$$\omega_o^2 = \frac{1}{LC} = \frac{K}{m}$$

$\omega = \omega_o k$ where ω_o is the resonant frequency.

$$\omega^2 L = \omega_o^2 k^2 L \text{ but } \omega_o^2 L = \frac{1}{C}$$

therefore $\omega^2 L = \dfrac{k^2}{C}$

If $R_2 = n_s R_0$ and $R_1 + R_2 = n_v R_0$, where R_0 is the resistance required to make the electrical circuit, made up of the resistances R_1 and R_2 and of the condenser and induction dead beat, then—

$$\omega R_2 = k\omega_0 R_2 = kn_s R_0 \omega_0 \text{ but } R_0\omega_0 = \frac{2}{C}$$

therefore $\omega R_2 = 2kn_s/C$ and $\omega(R_1 + R_2) = 2kn_v/C$

$$\therefore \sqrt{\frac{\left(\frac{1}{C} - \omega^2 L\right)^2 + (\omega R_2)^2}{\left\{\left(\frac{1}{C} - \omega^2 L\right)^2 + \omega^2(R_1 + R_2)^2\right\}\left\{k^4 + k^2(4n^2 - 2) + 1\right\}}}$$

$$= \sqrt{\frac{\left(\frac{1}{C}\right)^2 (1 - k^2)^2 + \left(\frac{1}{C}\right)^2 (2kn_s)^2}{\left\{\left(\frac{1}{C}\right)^2 (1 - k^2)^2 + \left(\frac{1}{C}\right)^2 (2kn_v)^2\right\}\left\{k^4 + k^2(4n^2 - 2) + 1\right\}}}$$

$$= \sqrt{\frac{(1 - k^2)^2 + (2kn_s)^2}{\left\{(1 - k^2)^2 + (2kn_v)^2\right\}\left\{k^4 + k^2(4n^2 - 2) + 1\right\}}}$$

$$= \sqrt{\frac{k^4 + k^2(4n_s^2 - 2) + 1}{\left\{k^4 + k^2(4n_v^2 - 2) + 1\right\}\left\{k^4 + k^2(4n^2 - 2) + 1\right\}}}$$

$$= \frac{1}{\sqrt{k^4 + k^2(4n_v^2 - 2) + 1}} \qquad \text{when } n_s = n.$$

i.e. when R_2 is in the same ratio to the resistance R_0 that the actual damping in the oscillograph is to the damping required to make it dead beat.

It is seen therefore that if the frequency at resonance of the electrical circuit is made equal to the frequency at resonance of the oscillograph, and if the ratio of the damping present in the shunt circuit to the damping *required for dead beat conditions is equal to the ratio of the damping present in the oscillograph to that required for dead beat conditions, then the actual magnification when shunted is given by*

$$\frac{1}{\sqrt{k^4 + k^2(4n_y^2 - 2) + 1}}$$

which is exactly the same expression as for oil damping (*see* p. 84).

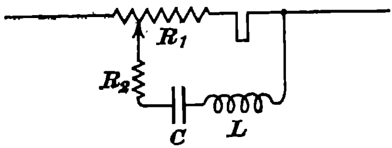


FIG. 54.—CONNECTION DIAGRAM FOR IRWIN RESONANT SHUNT TO PRODUCE VARIABLE DAMPING

But n_y, which is the ratio of $R_1 + R_2$ to R_o, replaces n which was the ratio of ρ to ρ_o.

If $n_y = 1$, i.e. if the value of $R_1 + R_2 = R_o$, where $R_o = 2L\omega_o = \frac{2}{\omega_o C}$, *then the oscillograph as shunted gives the same magnification as if it were damped by a viscous weightless fluid so as to be dead beat.*

If $n_y = \frac{1}{2}$, the damping is half critical.

If $n_y = 2$, the damping is twice critical.

Therefore, as the shunt circuit (which remains unchanged) can be arranged across a greater or lesser resistance in the main circuit, so the damping can be increased or diminished as shown in Fig. 54 where the value of R_1 can be altered by the slide.

It would be quite practical to have this slide wire graduated to show the value of the damping, or to indicate the resistance necessary to make the magnification unity for any particular harmonic. Thus, if the harmonic had a frequency equal to the resonant

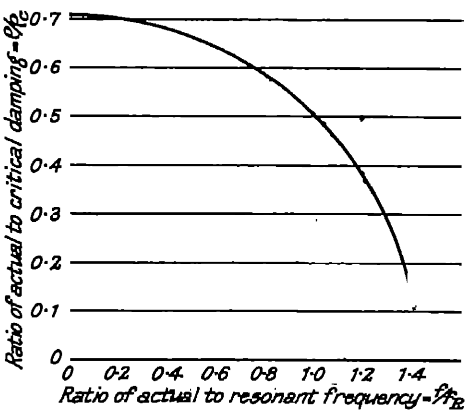


Fig. 55.—Curve Showing Damping Required for Unity Magnification, at Various Values of Frequency

frequency of the oscillograph, the value of $(R_1 + R_2)/R_0$ would have to be 0·5 (*see* Fig. 48, p. 83) to make the magnification unity.

In Fig. 55 a curve (a quarter of a circle to the scales chosen) is drawn to show the relation between the necessary damping and the frequency to make the magnification unity. The damping varies from 0·705 of critical at zero frequency to 0 at 41 per cent above the resonant frequency.

The phase difference between the current in the mains and the deflection of the oscillograph is shown in

Fig. 56; where i_s is the current in the shunt; i_m the current in the mains; and i_o the current in the oscillograph.

Since the resonant shunt circuit must be identical, as far as damping and phase displacement are concerned, with the oscillograph itself, we can replace K

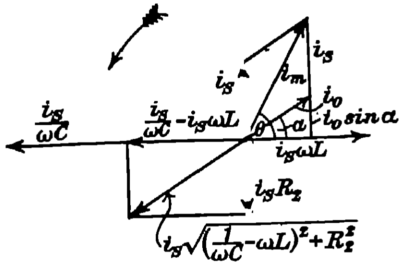


FIG. 56.—VECTOR DIAGRAM RELATING TO THE USE OF THE IRWIN RESONANT SHUNT

by $1/C$; ρ by R_2; and m by L as shown. The angle θ gives the angle of lag of the oscillograph deflection behind the current in the main, and a is the lag of the deflection behind the current in the oscillograph.

Now from the figure it will be seen that—

$$\tan\theta = \frac{i_s + i_o \sin a}{i_o \cos a} = \frac{i_s}{i_o \cos a} + \tan a$$

$$= \frac{i_s}{\dfrac{i_s\sqrt{\left(\dfrac{1}{\omega C} - \omega L\right)^2 + R_2{}^2}}{R_1}\cos a} + \frac{R_2}{\dfrac{1}{\omega C} - \omega L}$$

$$\text{but } \cos a = \frac{\dfrac{1}{\omega C} - \omega L}{\sqrt{\left(\dfrac{1}{\omega C} - \omega L\right)^2 + R_2{}^2}}$$

therefore $\tan\theta = \dfrac{R_1}{\dfrac{1}{\omega C} - \omega L} + \dfrac{R_2}{\dfrac{1}{\omega C} - \omega L}$

$$= (R_1 + R_2)\left(\frac{1}{\dfrac{1}{\omega C} - \omega L}\right)$$

Now $\omega C = k\omega_o C$; $\omega L = k\omega_o L$; and $\dfrac{1}{\omega_o C} = \omega_o L$

therefore $$\tan\theta = \frac{R_1 + R_2}{\omega_o L}\left(\frac{k}{1-k^2}\right)$$

and $$\omega_o L = \frac{R_o}{2}$$

therefore $\tan\theta = \dfrac{2(R_1 + R_2)}{R_o}\left(\dfrac{k}{1-k^2}\right) = 2n_v\dfrac{k}{1-k^2}$

where n_v is the ratio of actual damping to critical in the electrical circuit. This expression for determining the angle lag of the oscillograph deflection behind the current in the main circuit is exactly the same as that for the instrument with oil damping (*see* p. 86).

It is evident, therefore, that with the resonant shunt both the magnification and the phase displacement for any frequency are the same as for a massless viscous damping medium.

If both the magnification and the phase displacement are the same for the resonant shunt method as for the oil damping for every frequency, then they will give the same record for any shape of wave, i.e. *the resonant shunt can make the instrument dead beat for a suddenly applied pressure or for a rectangular wave.*

The direct proof for this was given to the author by Mr. T. Hodgson of the City and Guilds (Engineering) College, and as this confirms the results obtained by considering periodic functions, the author has pleasure in reproducing it below and of acknowledging at the same time the valuable help he received from Mr.

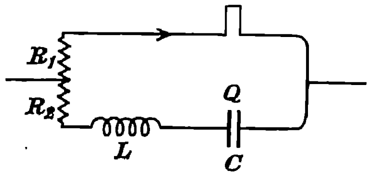


FIG. 57.—DIAGRAMMATIC REPRESENTATION OF RESONANT-SHUNT DAMPING

Hodgson in connection with the method. It was Mr. Hodgson who first showed that the values of R_1 and R_2, which are definite at the resonant point, also apply at all other frequencies.

Mr. Hodgson's Proof for Resonance Shunt Method. Referring to Fig. 57, which shows the circuit diagrammatically, we have the following basic equations—

Mechanical Oscillations.

$$m\ddot{\Delta} + \rho\dot{\Delta} + K\Delta = 0$$

$$\ddot{\Delta} + \frac{\rho}{m}\dot{\Delta} + \frac{K}{m}\Delta = 0$$

substituting $\frac{\rho}{m} = 2b$ and $\frac{K}{m} = \omega^2$

$$\ddot{\Delta} + 2b\dot{\Delta} + \omega^2\Delta = 0$$

and $\Delta = e^{-bt}(A \cos qt + B \sin qt)$ where $q^2 = \omega^2 - b^2$

Electrical Oscillations.

$$L\ddot{Q} + (R_1 + R_2)\dot{Q} + \frac{1}{C}Q = 0$$

$$\ddot{Q} + 2\lambda\dot{Q} + \omega^2 Q = 0$$

where $2\lambda = \frac{R_1 + R_2}{L} \qquad \frac{1}{LC} = \omega^2$

Case I. In this case, $R_1 + R_2 <$ critical resistance; $\lambda < \omega$; and the solution is of the form.

$$Q = e^{-\lambda t}\{A^1 \cos \mu t + B^1 \sin \mu t\}$$

where $\mu^2 = \omega^2 - \lambda^2$

Taking $Q = Q_o$, $\dot{Q} = 0$ when $t = 0$

I.e. In Fig. 57 consider that there has been a steady current flowing in the mains and that there is a charge Q_o in the condenser C, and that this current is broken at time $t = 0$.

$$A^1 = Q_o,\ B^1 = \frac{\lambda}{\mu}Q_o$$

whence $\dot{Q} = -\frac{\omega^2}{\mu}Q_o e^{-\lambda t}\sin \mu t$

$$i = -\dot{Q} = \frac{\omega^2}{\mu}Q_o e^{-\lambda t}\sin \mu t$$

where i is the current from the condenser that flows through oscillograph.

Due to i the motion of the oscillograph is given by

$$\ddot{\Delta} + 2b\dot{\Delta} + \omega^2\Delta = \frac{T\omega^4}{\mu}Q_o e^{-\lambda t}\sin \mu t$$

8—(5397)

Where T is a constant such that deflection $= T \times$ current. The solution of this expression is—

$$\Delta = e^{-bt} \{A \cos qt + B \sin qt\} + \frac{T\omega^2}{2\mu(\lambda-b)} Q_o e^{-\lambda t} (\lambda \sin \mu t + \mu \cos \mu t)$$

taking $\Delta = a$, $\dot{\Delta} = 0$ when $t = 0$

$B = 0$, and $A = 0$ if $a = \dfrac{T\omega^2}{2\mu(\lambda-b)} Q_o\mu$

i.e. if $i = \dfrac{Q_o\omega^2}{2(\lambda-b)}$

Now $Q_o = CR_1 i$

$$\therefore \frac{Q_o}{i} = CR_1 = \frac{2(\lambda-b)}{\omega^2} = 2LC(\lambda-b)$$

$$\therefore R_1 = 2L(\lambda-b)$$

but $R_1 + R_2 = 2L\lambda$ (from the definition of λ)

$$\therefore R_2 = 2Lb$$

$$\frac{R_2}{R_1 + R_2} = \frac{b}{\lambda}$$

which is the necessary condition for exact annulment of the free vibrations of the oscillograph by the current flowing from the condenser.

Case II. In this case $R_1 + R_2 >$ critical resistance; $\lambda > \omega$; $\lambda^2 - \omega^2 = \mu^2$; and—

$\Delta = e^{-bt} (A \cos qt + B \sin qt) + \dfrac{T\omega^2}{2\mu(\lambda-b)} Q_o e^{-\lambda t}$ ($\lambda \sinh \mu t + \mu \cosh \mu t$). A and B will be $= 0$ or there will be complete annulment if $i = \dfrac{Q_o\omega^2}{2(\lambda-b)}$ as before

which again leads to $R_2 = 2Lb$ and $\dfrac{R_2}{R_1 + R_2} = \dfrac{b}{\lambda}$

If, therefore, an adjustment be found for any one value of R_1 the value of R_2 will remain the same for all values of R_1.

For state of dead beat $\dfrac{R_2}{R_1 + R_2} = \dfrac{b}{\omega}$

The value of b can also be found directly. The period of the oscillograph being $\dfrac{2\pi}{q}$ or $\dfrac{2\pi}{\omega}$ approximately—

The ratio of successive amplitudes $= \dfrac{a_2}{a_1} = \varepsilon^{-b\frac{2\pi}{\omega}}$

therefore $b\dfrac{2\pi}{\omega} = \log\dfrac{a_1}{a_2}$

$$b = \frac{\omega}{2\pi}\log\left(\frac{a_1}{a_2}\right) = \text{frequency} \times \log\frac{a_1}{a_2}$$

b being known $R_2 = 2Lb$

and R_2 is now fixed for all variations of R_1.

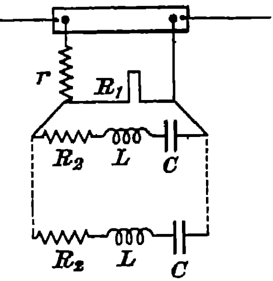
Up to the present we have been considering cases where the only shunt to the oscillograph is the resonant shunt. Where the instrument is used to record large currents the shunt in the main circuit is also shunting the oscillograph as shown in Fig. 58 and, as the resistance of this shunt is almost always negligible compared with that of the oscillograph, we can neglect it in considering the discharge of the condenser C through the oscillograph and the shunt in parallel. If an additional constant resistance r be placed between the main shunt and the oscillograph, a portion $r/(r + R_1)$ of the current from the condenser flows through the oscillograph, therefore we have to increase the capacity to $(r + R_1)/r$ of what it was before. Then, to keep the frequency constant, the induction L becomes $r/(r + R_1)$

what it was before. Also, R_2 becomes R_2^1, where $= R_2 \times r/(r + R_1)$. Now R_1 and r in parallel give esistance to the condenser discharge $R_1 \times r/(r + R_1)$, that the total resistance to the discharge of the con-

nser is $R_2^1 + R_1 \frac{r}{(r + R_1)}$ which is $\frac{r}{r + R_1}$ of the

ue of $R_1 + R_2$.

If, as practical values, $r = R_1$ then the capacity, inductance, and resistance would be altered in the correct proportions if an exactly similar resonance shunt were placed in parallel with R_2, C, and L, and connected as shown by the dotted lines in Fig. 58. The same result would be attained if the capacity were doubled and the resistance R_2 and the inductance L were halved. The damping



. 58.—Oscillograph, with sonant Shunt, connected ross Shunt in Main Circuit give the Wave Form of the Main Current

uld vary, however, if the value of r were altered to ry the magnitude of the deflection ; thus, if r were ried from $0{\cdot}5r$ to $1{\cdot}5r$ (where r is the value giving tical damping) the damping would be varied from 36 of critical damping to 1·2 time critical, which ould be less than the variation produced by oil mping owing to variation of viscosity with temper- ure.

Resonant Shunt Damping for Electrostatic Oscillo- aphs. This method of damping an electro-magnetic oillograph can also be applied to electrostatic instruments. Thus, in Fig. 59, if an electrostatic oscillograph *E.O.* were placed in parallel with the electromagnetic oscillograph, the electrical forces applied to the moving part of the electrostatic oscillograph would be proportional at every instant to the forces applied to the electro-magnetic instrument so long as the resistance R_1 was truly non-inductive and had no appreciable back-E.M.F. induced in it.

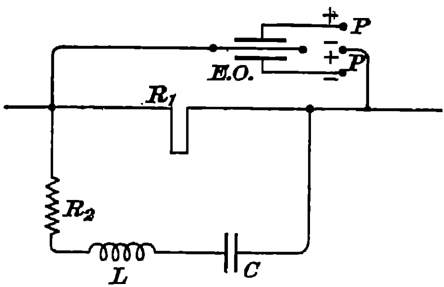


Fig. 59.—Resonant-shunt Damping Applied to an Electrostatic Oscillograph

If, therefore, the electrostatic oscillograph be made of the same frequency as the electro-magnet instrument, and has the same ratio of actual damping to critical damping, it can be made dead beat, under-damped, or over-damped by varying R_1, just as in the case of the electro-magnetic oscillograph. In practice, R_1 would be made a very high resistance and this means that R_2 and L would also be very large, and the capacity C would be very small.

Let us consider the case of an electrostatic oscillograph of a capacity $40 \times 10^{6}\ \mu F$, with a resonant frequency of 2000, and suppose that the damping out of oil is one-twentieth of the critical value.

At its own natural frequency, if the voltage required to give a suitable deflection were 2000 V, the capacity current taken by the instrument would be—

$$\omega_0 CV = 1{,}260{,}000 \times 40 \times 10^{-12} = 50 \times 10^{-6}\text{ A.}$$

If the current through the resistance R_1, were, say, ten times as great, then $R_1 = 2000/(50 \times 10^{-5}) = 4 \times 10^6 = 4$ megohms.

If one required to use this oscillograph to give a record of the voltage on, say, a high pressure system

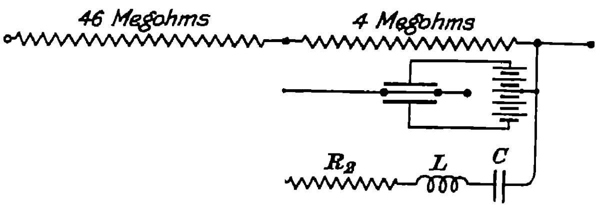


FIG. 60.—ELECTROSTATIC OSCILLOGRAPH WITH RESONANT-SHUNT ARRANGED TO RECORD EXTRA-HIGH VOLTAGE

of 33,000 V, the series resistance would be of the order of 50 megohms, and the current taken from the mains would be of the order of 0·5 mA. The arrangement would be as shown diagrammatically in Fig. 60.

To make the instrument dead beat

$$\frac{R_2}{4 \times 10^6 + R_2} = \frac{1}{20}$$

therefore $R_2 = 4 \times 10^6/21 = 200{,}000$ ohms (approx).

Now $R_1 + R_2 = \dfrac{2}{\omega_0 C}$ (*see* p. 96),

therefore $C = 2/(4 \times 10^6 \times 12{,}600) = 4 \times 10^{-5} \mu\text{F}$,

and $\quad 2\omega L = R_1 + R_2 = 4 \times 10^6$ ohms

therefore $L = 4 \times 10^6/12{,}600 = 317$ henries a value which, although high, is fairly easily attainable with the fine wire now available.

The advantage of a resonant shunt for damping an electrostatic oscillograph is that it makes the damping independent of the dielectric medium, which can therefore be chosen for reasons other than damping. Thus, compressed air, or any gas, could be chosen instead of

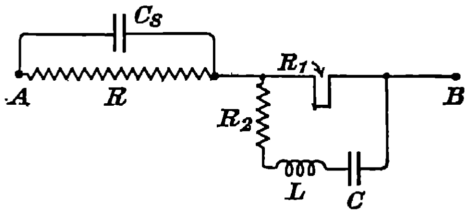


FIG. 61.—RESONANT SHUNT USED WITH ELECTROMAGNETIC OSCILLOGRAPH, AND WITH A CONDENSER SHUNTING THE SERIES RESISTANCE TO INCREASE THE RANGE OF FREQUENCY OVER WHICH THE INSTRUMENT CAN BE USED

oil. If a very high vacuum be used the dielectric strength can be increased to, say, twenty times that of air or, say, seven to ten times that of oil, and this will make possible frequencies for electrostatic instruments as high as for electro-magnetic.

Working Range of Frequency with Resonant-shunt Damping. Since the resonant shunt method of damping gives a means of producing less than or more than critical damping at will, it opens up a new method of increasing the range of frequency over which an oscillograph can be used.

If an oscillograph of the electro-magnetic type be used to indicate the pressure between two points AB, Fig. 61, and if the value of R_1 be adjusted to give, say, twice critical damping the curve of magnification for twice critical damping given in Fig. 51 would apply to the instrument and, at its own resonant frequency, the amplitude of the vibrations would only be one quarter of the amplitude given by the same pressure variation at very low frequencies.

If, however, the series resistance R be shunted by a capacity C_s the magnitude of the current taken from the mains when R is very large compared with R_1 becomes $E\sqrt{(\omega C_s)^2 + \left(\frac{1}{R}\right)^2}$ instead of $\frac{E}{R}$, therefore, if the ratio of ω to $\omega_o = k$ as before the ordinary magnification of amplitude for an oscillograph

$$\frac{1}{\sqrt{k^4 + k^2(4n^2 - 2) + 1}} \text{ (p. 84)}$$

becomes—

$$\frac{1}{\sqrt{k^4 + k^2(4n^2 - 2) + 1}} \left[\frac{\sqrt{(k\omega_o C_s)^2 + \left(\frac{1}{R}\right)^2}}{(1/R)}\right]$$

since $\omega = k\omega_o$.

If the ratio of $\omega_o C_s$ to $1/R$, i.e. the ratio of the current taken by the condenser C_s to the current taken by the series resistance R at the resonant frequency of the oscillograph, be called q

Then the magnification $= \sqrt{\frac{(kq)^2 + 1}{k^4 + k^2(4n^2 - 2) + 1}}$

If n, the ratio of damping to critical damping $= 2$, and if we make k the ratio of frequency to resonant frequency $= 1$, the magnification $= \sqrt{\frac{q^2 + 1}{16}}$

Thus, at the resonant frequency, if the magnification = 1

$$q^2 + 1 = 16$$
$$\text{and } q = 3{\cdot}87$$

If $q = 4$ the magnification at resonant frequency $= 1{\cdot}03$

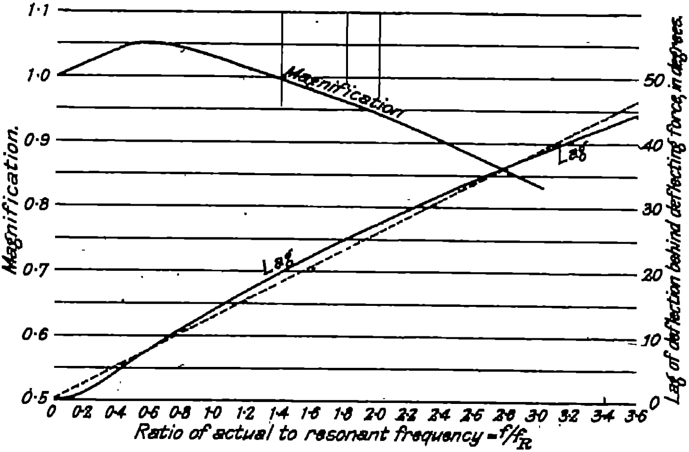


FIG. 62.—CURVES SHOWING MAGNIFICATION AND LAG OF OSCILLOGRAPH DEFLECTION AT VARIOUS FREQUENCIES WITH TWICE CRITICAL DAMPING, AND WITH A CONDENSER SHUNTING THE SERIES RESISTANCE

If we take $q = 4$ and $n = 2$, and plot the values of $\sqrt{\dfrac{16k^2 + 1}{k^4 + k^2(4n^2 - 2) + 1}}$ from $k = 0$ to $k = 3$ we get a curve as shown in Fig. 62.

It will be seen that the variation of magnification (positive or negative) nowhere exceeds 5·5 per cent over a range of frequency from 0 to twice the resonant frequency of the instrument and that even at three times the resonant frequency the reduction of amplitude is only 16·5 per cent, whereas the same instrument critically damped and used at three times its resonant frequency would have a reduction of amplitude of 90 per cent.

The phase displacement a of a damped oscillograph working in the usual manner is given by—

$$\tan a = \frac{2nk}{1-k^2} \quad \text{(p. 86).}$$

but if the series resistance R be shunted by a condenser the current from the mains will lead in the pressure by the angle $\tan^{-1} \frac{\omega C}{1/R}$ but $\omega = \omega_0 k$ and $\frac{\omega_0 C}{1/R} = q$ therefore $\tan^{-1} \omega CR = \tan^{-1} qk$.

The angle between the applied pressure and the oscillograph deflection is therefore—

$$\tan^{-1} \frac{2nk}{1-k^2} - \tan^{-1} qk.$$

The angle $\tan^{-1} \frac{2nk}{1-k^2}$ is already plotted on the curves shown in Figs. 49 and 51, and we have only to subtract $\tan^{-1} qk$ from the angles there shown.

For $q = 4$ and $n = 2$ a phase displacement curve is shown in Fig. 62 whence it will be seen that the phase displacement is practically a straight line over a range up to 3·5 times the resonant frequency.

The actual curve is nowhere more than 2° from the straight line given by $13k$.

Recording Transient Phenomena. Using a condenser across the series resistance of the oscillograph. Since this method of working an oscillograph shows such marked advantages over the usual method for periodic variations it is necessary to inquire into the performance of the instrument when subject to transient phenomena.

It is necessary to consider the order of magnitude of the various portions of the electrical circuit shown in Fig. 61. If R_1 be the resistance of the oscillograph this can be made, say, of the order of 5 ohms.

If a current of 0·1 A gives a large enough deflection then, on a voltage of 100, the series resistance R is 1000 ohms. If the oscillograph have a resonant frequency of 4000 then in the above case, where $\omega C_s R = 2$

then $C_s = \dfrac{2}{1000 \times 2\pi \times 4000} = \dfrac{1}{4\pi}\,\mu F$.

If a steady voltage V be applied suddenly across the terminals AB (Fig. 61), the condenser C_s is charged through the resistance R_1 of the oscillograph and the charge Q in the condenser at any time t from the application if the voltage is given, in terms of the final charge VC_s, by the equation—

$Q = VC_s(1 - \varepsilon^{-t/C_sR_1})$.

In the present case, $C_sR_1 = \dfrac{\cdot 1}{4\pi} \times 10^{-6} \times 5 = 4 \times 10^{-7}$

and if $t = 4 \times 10^{-7}$ second, then $(1 - \varepsilon^{-t/C_sR_1}) = 0{\cdot}63$ i.e. the condenser acquires 63 per cent of its charge in less than half a millionth of a second.

In one-millionth of a second the condenser acquires about 92 per cent of its complete charge, that is to say in a time very short compared with the frequency of the oscillograph the charge C_sV will flow into the condenser. This charge will give an impulse to the oscillograph strips per unit length, equal to $C_s(V/10)B$, where B is the strength of the magnetic field in which the strips are placed. If m be the mass per unit length, then the initial velocity is given by $C_s \frac{V}{10} \frac{B}{m}$ or, say, $\frac{\gamma C_s V}{m}$.

If the resistance of the oscillograph were lower, or if the value of $\omega C_s R$ were less than 2, then this velocity would be attained in a still shorter time.

To consider how the initial velocity $\frac{\gamma C_s V}{m}$ affects the oscillograph let us take first the simple case of a critically damped oscillograph, i.e. the case in which $\rho = 2\omega m$. In Fig. 63 the lower curve shows the deflection of a critically damped instrument plotted against an arbitrary scale of time when a constant force is suddenly applied to the moving element. In the present case it is the current due to a voltage V.

At first sight it would not appear that, when an instrument is just dead beat or critically damped, one could give it an initial velocity and yet, with the same applied force, still have the instrument dead beat, i.e. have no over-shoot of the final position. That this is so, however, is evident if we consider the upper curve in Fig. 63, which shows the deflection plotted against time for an applied voltage $2V$. This curve has at every time just twice the amplitude of the lower curve. When the deflection for the curve for $2V$ attains to the final deflection for V the resultant force due to the difference between the current in the strips and the control force is just equal to the force on the strips at the beginning of the curve for V. It is obvious, therefore, that if tan a be the rate of change of the curve for $2V$, at the moment when this curve has half its final value, the curve for V could *start* with a slope at least equal to the angle a. The curve for deflection against time for applied voltage V would then be $a^1b^1c^1d^1$ exactly similar to the portion of the curve $abcd$ for $2V$. Also, if the curve $abcd$ be tangential to its final position then $a^1b^1c^1d^1$ will be tangential to

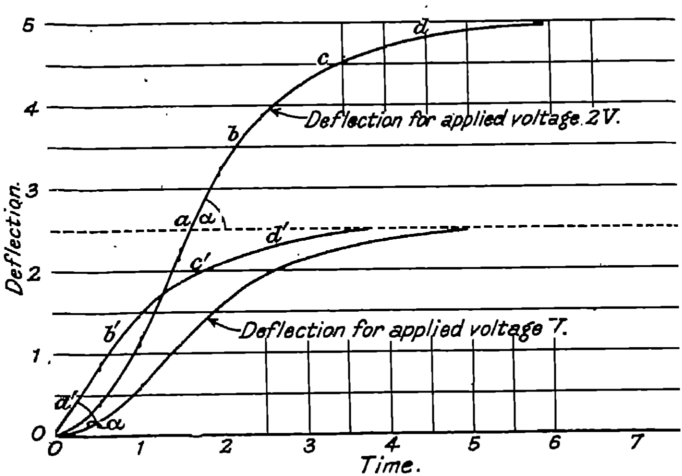


FIG. 63.—CURVES SHOWING RELATION BETWEEN DEFLECTION AND TIME WITH CRITICAL DAMPING

(*Note.*—The scales of deflection and time are arbitrary)

its final position, i.e. it will not cross or overshoot the line corresponding to its final value and so is still dead beat.

Fig. 63 shows, especially for the initial portions of the curve, how much more quickly the curve starting with an initial velocity attains to a particular value as compared with the ordinary damping curve; thus one-fifth of the final deflection is attained in one-third of the time and one-half of the final deflection in half the time. (These values are approximate as they are measured off the curve).

Mr. Hodgson has worked out the general case for a critically damped oscillograph as follows—

$$\mathrm{m}\ddot{\Delta} + \rho\dot{\Delta} + K\Delta = \gamma\frac{V}{R}$$

therefore $\ddot{\Delta} + \frac{\rho}{m}\dot{\Delta} + \frac{K}{m}\Delta = \frac{\gamma}{m}\frac{V}{R}$

where γ is the electro-magnet force for one ampere.

Let $\frac{K}{m} = \omega^2$. Final displacement $\Delta = \frac{\gamma V}{m\omega^2 R} = a$

Initial velocity $\dot{\Delta} = \frac{\gamma C_s V}{m} = u$.

For critical damping $\frac{\rho}{m} = 2\omega$

$\ddot{\Delta} + 2\omega\dot{\Delta} + \omega^2\Delta = \omega^2 a$

solution $\Delta = a + (A + Bt)e^{-\omega t}$

then $\dot{\Delta} = (B - A\omega - B\omega t)e^{-\omega t}$

when $t = o$ $\Delta = o$ $\therefore$ $A = -a$

and when $t = o$ $\dot{\Delta} = u$ $\therefore$ $u = B + a\omega$ $B = u - a\omega$

For maximum deflection—

$\dot{\Delta} = o$ $\therefore$ $B - A\omega - B\omega t = o$ or $t = \frac{1}{\omega} - \frac{A}{B}$

or $e^{-\omega t} = o$, i.e. $t = \infty$

overshoot $\Delta - a = (A + Bt)e^{-\omega t}$ for this value of t,

" $= o$ if $A + Bt = o$, i.e. $t = -\frac{A}{B}$

or $e^{-\omega t} = o$, i.e. $t = \infty$.

For these two values of t to agree $\frac{1}{\omega} = o$ $\therefore$ $\omega = \infty$; i.e. there is no possible solution if C_s does not equal zero, since the alternative is $e^{-\omega t} = o$, i.e. $t = \infty$.

If C_s does not equal zero there will never be any overshoot since—

$$t = \frac{1}{\omega} - \frac{A}{B} = \frac{1}{\omega} - \frac{a}{a\omega - u} = \frac{-u}{\omega(a\omega - u)} \text{ is negative}$$

unless $u > a\omega$, i.e. $C_s > 1/R\omega$, i.e. $C_sR\omega > 1$

It follows from this that the instrument can be given an initial velocity $u = a\omega$, or $C_sR\omega$ can be made equal to one, and yet the instrument will be dead beat.

If $C_sR\omega$ be greater than one then there is overshoot and Mr. Hodgson gives the solution as follows—

Maximum deflection occurs when $t = \frac{1}{\omega} + \frac{a}{u - a\omega} = \frac{u}{\omega(u - a\omega)}$

$$= \frac{C_s}{\omega\left(C_s - \frac{1}{R\omega}\right)} = \frac{RC_s}{RC_s\omega - 1}$$

$$\text{Overshoot} = \{-a + (u - \omega a)\, t\}e^{-\omega t}$$

$$= \left\{-a + \frac{u}{\omega}\right\} e^{-\omega t} = \frac{u - a\omega}{\omega} e^{-\omega t}$$

$$= \left(\frac{\gamma C_s V}{m\omega} - \frac{\gamma V}{m\omega^2 R}\right) e^{-\frac{RC_s\omega}{RC_s\omega - 1}}$$

$$= \frac{\gamma V}{m\omega^2 R}(RC_s\omega - 1)\, e^{-\frac{RC_s\omega}{RC_s\omega - 1}}$$

since $\frac{\gamma V}{m\omega^2 R} = a.$

$$\text{Ratio of overshoot to } a = (RC_s\omega - 1)\, e^{-\frac{RC_s\omega}{RC_s\omega - 1}}$$

$$= (q - 1)\, e^{-\frac{q}{q-1}}$$

where $q = RC_s\omega$ as before.

The following table shows the ratio of overshoot to final deflection for various values of $RC_s\omega$, i.e. of q—

$RC_s\omega$	1·2	1·4	1·6	1·8	2·0	2·2	2·4	2·6
Ratio of Overshoot to Final Deflection	·0005	·012	·041	·085	·136	·195	·25	·31

It is seen that for values of $RC_s\omega = 2$ and $n = 1$, the overshoot is only 13·6 per cent of the final steady deflection.

It is shown on p. 108 that the magnification of the oscillograph when critically damped and with the series resistance R shunted by a condenser, is given by

$$\sqrt{\frac{(kq)^2 + 1}{k^4 + k^2(4n^2 - 2) + 1}};$$

where q is the value of $RC_s\omega$; k = ratio of the actual frequency to the resonant frequency of the oscillograph; and n = ratio of actual to critical damping.

If $n = 1$ and $k = 1$, i.e. in the case of an oscillograph critically damped and working at its resonant frequency, the magnification $= \sqrt{[(q^2 + 1)/4]}$ so that, if $q = 1$, the magnification = 0·7.

If we wish to make the magnification = 1, then $q^2 + 1 = 4$ and $q = 1{\cdot}73$. For this value of q the overshoot of the deflection is only 6 per cent.

The variation of magnification with applied frequency, when working with a value of $q = 1{\cdot}73$ and with critical damping, is shown in Fig. 51 (p. 87) to make it comparable with the instrument as used normally. From this curve it is seen that the instrument will work over a range up to 1·3 times its resonant frequency with an error not greater than the instrument, as used normally, would have up to 0·3 of its resonant frequency. In other words the range of frequency on which the instrument can be used is made over four times as great as when used without the condenser shunting the series resistance.

The phase displacement for the conditions here considered ($q = 1{\cdot}73$ and critical damping) is also shown in Fig. 51 (lower dotted curve) whence it is seen that the displacement in degrees is practically proportional to the value of k ($= f/f_R$) and is very nearly expressed by $28k$. The chain dotted line in the figure is drawn for a displacement $= 28k$.

If the displacement were *exactly* proportional to k there would be no displacement of the various harmonics relative to the fundamental.

The above results hold for any electro-magnetic oscillograph, *no matter how damped*, therefore the range of frequency of any Duddell oscillograph damped in the usual manner could be increased fourfold by placing in parallel with the series resistance, a condenser taking a current 1·73 as great as the series resistance at a frequency equal to the resonant frequency of the oscillograph. The curve given by an instrument of nominal frequency 2500 would then be more accurate than one of nominal frequency 10,000 used in the ordinary way.

Mr. Hodgson has also found an expression for the maximum value that the condenser C_s can have, so that, when the damping is greater than critical, the instrument still gives a deflection which just fails to overshoot its final value, i.e. is just dead beat. The relation that he finds is—

$$\rho = m\omega\left(RC_s\omega + \frac{1}{RC_s\omega}\right)$$

but $\rho_0 = 2m\omega$ is the relation for critical damping.

$$\text{Therefore } \frac{\rho}{\rho_0} = \frac{RC_s\omega + \dfrac{1}{RC_s\omega}}{2}$$

or, denoting $\frac{\rho}{\rho_0}$ by n; and $RC_s\omega$ by q:—

$$2n = q + \frac{1}{q}.$$

On p. 109 there is a case worked out for $n = 2$ and $q = 4$, i.e. with $n = 2$ instead of $\frac{1}{2}(4 + \frac{1}{4}) = 2{\cdot}125$ which is the value necessary to give no overshoot, and the curve is shown in Fig. 62.

It is well known that, when the damping is very nearly critical, under a given condition of working, a comparatively large variation in ρ will not affect the value of the overshoot to any noticeable extent, therefore the oscillograph so arranged would show no appreciable overshoot.

Curves could be drawn for values of q plotted against n for critical damping and curves obtained similar to those in Fig. 62 for particular values of n and q.

For higher values of q, n can be taken as half q without any serious error, thus if $q = 10$ the value $n = 5$ is within 1 per cent of the true value.

In such a working of the oscillograph the control due to damping forces has been made very large compared with the inertia and control forces.

In the preceding cases the results were worked out for an electro-magnetic oscillograph used to indicate the pressure between two points in a circuit. The conditions are altered when the instrument is used to indicate the current flowing in a circuit and where the instrument is used as a shunt to a resistance. It is evident that the resistance must now have self-induction as well as resistance and, if the current and potential curves are to be exactly comparable, the ratio of the reaction to the resistance voltage must be equal to the ratio $\omega C_s/(1/R)$; i.e. $\omega L/R_s = \omega C_s R$; where L is the self induction of the resistance used as a shunt, and R_s its resistance.

It might be considered at first somewhat troublesome to adjust the resistances in the main circuit to have this value of $\omega L/R_s$ but, except for the resistance shunts for small currents, the resistance will have nearly enough self-induction. Thus, if $\omega C_s R = q = 2$, then for a shunt to give 1 V drop for a current of 50 A $\omega L/R_s = \omega C_s R = 2$; $R_s = 0{\cdot}02$; and $\omega L = 0{\cdot}04$. Hence, if $\omega = 2\pi$, $4000 = 25{,}000$ (i.e. if resonance frequency $= 4000$) $L = 1{\cdot}6$ microhenries.

A single turn of No. 14 S.W.G. wire, in the form of a square of 10 cm. side, would have an inductance of about 0·4 microhenry, and less than three such turns of wire would be required to give the necessary inductance. A simple mutual inductance could also be arranged to have the necessary ratio of M to R.

Practical Conclusions Regarding Range and Error of Oscillographs. To come to practical results in connection with the errors due to oscillographs it is sufficient at present, to say that an Einthoven galvanometer or oscillograph, in which there is no loading of the wire by a mirror, fulfils the above theory very exactly, but an oscillograph of the bifilar type does not conform so closely.

The one reason for this is that a bifilar instrument has generally two essential periods of vibration as it is almost impossible to arrange the two wires to have equal tensions, and to place the mirror absolutely symmetrically across the wires. There may also be a condition analagous to the case of two closely coupled circuits where the energy surges alternatively from one circuit to the other; in the present case the energy of vibration may surge fron one wire to the other owing to the coupling due to the mirror. These defects are not important in cases where the damping is large and only appear when the ratio of actual damping to critical damping is small.

The principal disadvantage of the bifilar oscillograph is the loading of the wire due to the mirror. The theory as far as we have given has been based on the assumption that the forces per unit length (due to the current, inertia, damping, and control) are uniform along the length of the wire but if there be considerable mass lumped in the middle of the wire it is evident that the damping force for unit velocity should be greater at this point. If it cannot be made greater locally, then we must make the damping of the strips greater than necessary to prevent overshoot when a voltage is suddenly applied. This is the condition under which, at present, all oscillographs of the bifilar type work when immersed in oil, and it accounts for the falling-off in deflection of the oscillograph with increase in the frequency of the current flowing through it. The fact that this falling-off is greater than the theoretical value shows that the instrument is really overdamped.

The curves shown in Fig. 66 (p. 123) show how impossible it is to make any theoretical deductions from an oil-damped oscillograph of the bifilar type, especially when neither the effective mass, nor the resonant frequency, can be determined with any degree of accuracy.

When one comes to apply the resonant shunt method to a bifilar oscillograph, the departure of the oscillograph from a simple vibrating system becomes apparent, and a resonant shunt circuit worked out for a particular instrument shows for a square wave of applied voltage a small residual ripple of the order of from one-twentieth to one-tenth of the deflection, thus showing that about nine-tenths of the deflection of

FIG. 64.—RECORDS OF THE SUDDEN APPLICATION OF A STEADY CURRENT TO A HIGH-FREQUENCY BIFILAR OSCILLOGRAPH (*a*) UNDAMPED, (*b*) WITH RESONANT-SHUNT DAMPING

the strips had been absorbed, or 99 per cent of the energy. If the vibrating system had been a simple vibrating system the absorption of energy would have been complete as shown in Fig. 17(*a*) for an Einthoven galvanometer. Even when there is an average residual deflection of 8 per cent with the resonant-shunt method, the interpretation of the results is easy as the resonant frequency of the oscillograph is generally much higher than the wave under observation.

Fig. 64, herewith, shows the result of sending a constant current through a Duddell high-frequency oscillograph. In the upper curve, with the strips undamped, there can be seen the effect of the beats between the two wires. At the moment of break owing to the extremely rapid movements of the spot the curve is very faint. In the lower curve the effect of using the oscillograph with a resonant shunt is

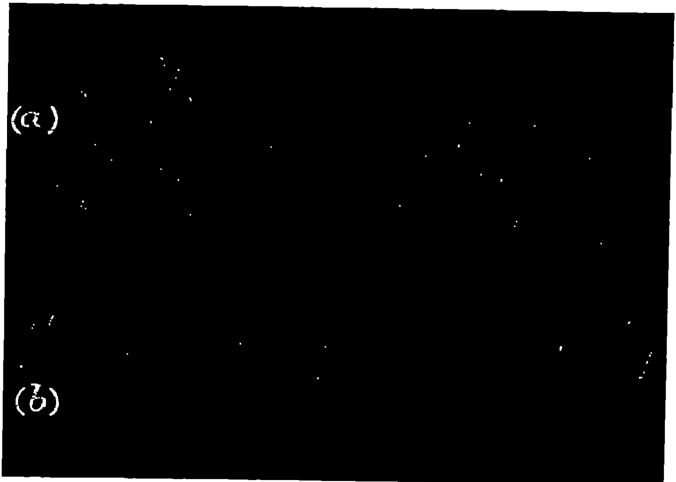


Fig. 65.—Records obtained with a High-frequency Bifilar Oscillograph Critically Damped (a) with Oil (b) by a Resonant Shunt

clearly shown, the general outline following the usual damping curve for an oscillograph, but showing small residual vibrations for reasons already explained. The maximum value of these vibrations is about 5 per cent of the initial displacement, and this value of 5 per cent is nearly twice what would be produced if there were no beats between the two wires. When taking this record Fig. 65(b) the oscillograph was over-damped in order to show the curve more clearly, but the amount of the damping can be varied easily at will.

Fig. 65 is reproduced from two oscillograph records taken on the same plate, showing the results obtained (*a*) by a high-frequency Duddell oscillograph critically damped by oil (upper curve); and (*b*) by the same type of oscillograph damped by a resonant shunt

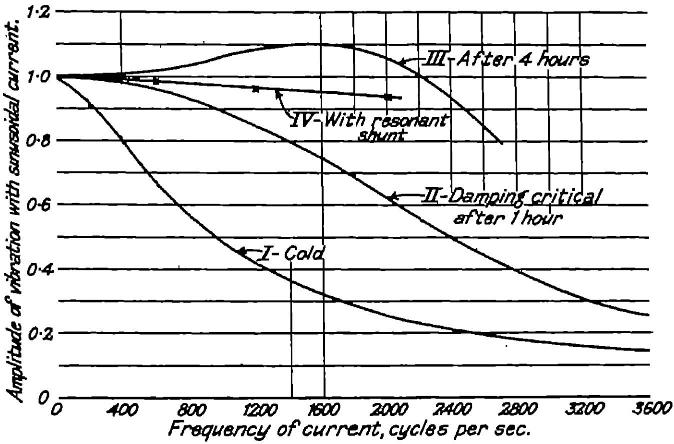


Fig. 66.—Curves Showing Relation between Amplitude of Vibration and Frequency of Current in a Bifilar Oscillograph with Oil Damping at Various Temperatures, and with Resonant-shunt Damping

(lower curve). In spite of the great irregularity in the curve the two records are almost exactly similar; actually the peaks are slightly higher with the resonant shunt. As the highest important harmonic in the record has a frequency not exceeding 1000, one would, of course, expect the records of both elements to be identical and true. It is only when the frequencies are much higher, and near the resonant frequency, that one observes a discrepancy.

In Fig. 66 is shown the performance under various conditions of a bifilar oscillograph made by the General Electric Co. (U.S.A.).

The oscillograph is one with a nominal frequency of about 4000 and its performance is shown under four conditions: (I) with the oscillograph cold; (II) with the oscillograph just showing critical damping; (III) with the oscillograph hot after four hours; and (IV) with a resonant shunt. The heating was due to the exciting circuit of the electro-magnet.

The curves show the relation between the amplitude of vibration for a sine wave of current and the frequency of this current. This corresponds to the magnification curves plotted in Fig. 51 against the ratio of frequency to resonant frequency.

Curve II for critical damping, i.e. just dead beat, shows an amplitude of deflection at frequency 2400 equal to half that given at frequency zero. If we deduce from this that the resonant frequency is 2400 it means that the presence of the oil has reduced the resonant frequency from 4000 to 2400 or made it 0·6 of what it would be out of oil. A usual value for this ratio is about 0·66.

The curve (I) showing the relation between frequency and amplitude with the oscillograph cold has, at a frequency 2400, an amplitude 0·21 of the amplitude at frequency zero, so that the damping when cold is 0·5/0·21 or 2·3 times the critical value. At a frequency 2400 with the oscillograph hot the amplitude is 0·93 and the damping is 0·5/0·93 or 0·54 of critical, so that, due to change of temperature alone, the deflection at that frequency can vary over a range of 4/1.

At half the resonance frequency, or 1200, the deflection can change from 0·42 to 1·08, i.e. over a range of 2·5/1.

On the other hand, the same instrument used with a resonant shunt, curve (IV), has up to 2000 frequency a maximum change of amplitude of 6 per cent, corresponding to a condition of damping between half and full critical damping, and it will remain constant at all temperatures.

The principal conclusions to be drawn from these results are—

(1) Using an oscillograph damped in the usual way: (*a*) it is advisable for the series resistance to have a capacity effect either inherent or added; and (*b*) the shunts in the main circuit should be inductive, *not* non-inductive.

(2) Using a resonant shunt for damping purposes has very considerable advantages as it does not reduce the resonant frequency of the oscillograph and the damping can be made adjustable and definite. In this case also: (*a*) there should be a capacity shunting the series resistance; and (*b*) the shunts in the main circuit should be made inductive.

(3) An electrostatic oscillograph can be damped by a resonant shunt and can be placed in a high vacuum to enable large electrical stresses to be employed.

CHAPTER IV

THE CATHODE RAY OSCILLOGRAPH

General Principle. The cathode ray oscillograph w first developed by Braun and is sometimes spoken of a Braun Tube. It depends on the principle that a stre of negative corpuscles can have their path deflected either an electro-magnetic or an electrostatic field.

If a highly exhausted glass tube such as shown Fig. 67 has a cathode plate K sealed into one end a a anode plate A some distance along the tube, the when a high enough voltage is applied between t plates K and A, an electrical discharge will pass a this discharge will be carried in the tube chiefly very minute bodies called electrons, each of whi carries a negative charge of electricity. Sometim the charge and its associated body is called an electrc

As the current is carried chiefly by negative charg these will be discharged against A with great velocit and if there is a pin hole in the centre of A some of t electrons will be projected through this opening dov a small tube t. In their flight they will pass: (between a pair of parallel plates PP with their pla horizontal; and (2) between a pair of plates QQ wi their plane vertical and will eventually strike t screen S and cause fluorescence.

An electrostatic field can be established between t plates PP or between the plates QQ; and a pair coils CC can produce a magnetic field superimposed the electrostatic field due to the plates QQ.

Dynamics of the Braun Tube. A body of mass m, carrying a charge e, measured in electrostatic units will be subject to a force in an electrostatic field given by eE, where E is the strength of field in electrostatic units, the direction of this force will be along the lines of force. If the body has motion at right angles to a magnetic field it will also have a force acting on it at right angles to its motion and to the magnetic field.

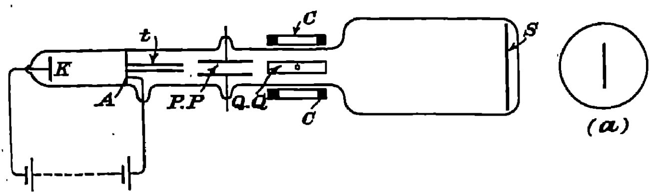


FIG. 67.—DIAGRAMMATIC REPRESENTATION OF THE BRAUN TUBE

Force and Motion in a Magnetic Field. In considering this case, let all measurements be made in electromagnetic units. A charge e moving at a velocity v is the equivalent of a current of ev and the force in dynes on a conductor carrying this current would be evB per unit length, where B is the intensity of the magnetic field.

If the moving charge has a length of path in the magnetic field $= \lambda$, the time during which the force evB acts is λ/v, and the displacement δ_1 from the initial straight line during this time will be

$$\delta_1 = \tfrac{1}{2}at^2 = \frac{evB}{2m}\,\frac{\lambda^2}{v^2}$$

The velocity impressed on the moving charge by the magnetic field is $\dfrac{evB}{m}\,\dfrac{\lambda}{v}$ so that if the charged body has a distance L to travel after it has passed out of the magnetic field it will have a further displacement $\delta_2 = \frac{evB\lambda}{mv}\frac{L}{v}$ or a total displacement $\delta = \delta_1 + \delta_2 = \frac{evB}{2m}\frac{\lambda^2}{v^2} + \frac{evB\lambda L}{mv^2} = \frac{e}{m}\frac{B\lambda}{v}\left(\frac{\lambda}{2} + L\right)$ *cms. where all measurements are in electro-magnetic units.*

Force and Motion in Electrostatic Field. In this case, all measurements are made in electrostatic units. If a

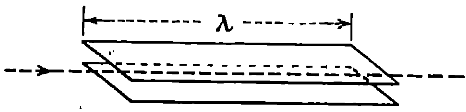


FIG. 68.—PATH OF A PARTICLE BETWEEN TWO PARALLEL PLATES

body of mass m and a charge e has a velocity v and if it pass in its flight between two parallel plates (Fig. 68), which are maintained at a difference of potential E, then, if the distance between the plates be d, the electric field is E/d and the force acting on the body is eE/d.

The time required to pass the plates is λ/v and the displacement δ_1 from the initial straight line

$$= \tfrac{1}{2}at^2 = \tfrac{1}{2}\frac{E}{dm}e\left(\frac{\lambda}{v}\right)^2.$$

The velocity impressed on the moving charge by the field is $\frac{E}{d}\frac{e}{m}\frac{\lambda}{v}$, so that if the charged body has a distance L to travel after it leaves the influence of the field it will have a further displacement $\delta_2 = \frac{E}{d}\frac{e}{m}\frac{\lambda}{v}\frac{L}{v}$.

Total displacement $\delta = \delta_1 + \delta_2 = \frac{E}{d}\frac{e}{m}\frac{\lambda}{v^2}\left(L + \frac{\lambda}{2}\right)$ $= \frac{E}{v^2}\frac{e}{m}\frac{\lambda}{d}\left(L + \frac{\lambda}{2}\right)$ *cms. where all measurements are in electrostatic units.* The value of e/m for cathode rays $=$ $5{\cdot}316 \times 10^{17}$ electrostatic units or $1{\cdot}772 \times 10^7$ electromagnetic units.

The velocity v given to the electrons is determined by the voltage between the cathode and anode plates. If we keep in mind the fact that the work done on a body, in moving it from a plate at one potential to another at a different potential, is independent of the path taken and depends only on the charge on the body it is easy to calculate the velocity of the moving corpuscles.

If the distance between the cathode and anode plates is D, the field in electrostatic units is $\frac{V}{D}$. The force on the electron is $\frac{Ve}{D}$ the acceleration $= \frac{Ve}{Dm}$ and

$$v^2 = 2 \times \text{acceleration} \times D = 2\frac{V}{D}\frac{e}{m}D = 2V\frac{e}{m}$$

therefore $v = \sqrt{2V\frac{e}{m}}$.

If $\frac{e}{m} = 5{\cdot}316 \times 10^{17}$

$$v = 10{\cdot}3 \times 10^8\sqrt{V}$$

If V_1 be measured in volts then

$$v = 10{\cdot}3 \times 10^8\sqrt{\frac{V_1}{300}} = 5{\cdot}95 \times 10^7\ \sqrt{V_1}.$$

This assumes that no energy is spent in tearing away the electrons from the cathode, or absorbed due to viscosity in the medium through which they pass.

Sensitivity of the Braun Tube. We are now in a position to calculate the sensitivity under any given condition.

In the tube shown in Fig. 67 when a stream of electrons is projected at a high velocity against *A* some of them will pass through a small aperture in the centre of the plate. A diaphragm with a hole in the centre, or else a pinhole tube as shown at *t*, Fig. 67, is interposed between the anode *A* and the screen *S* on which the stream of electrons (or " cathode ray ") falls ; this is to cut off secondary radiation and light, and to confine the pencil of rays and the spot to a small diameter.

Case (*a*)—*Electro-magnetic Operation.* If the cathode ray, after passing the diaphragm, pass into a magnetic field of uniform strength and if the strength is uniform for a distance of 10 cm. and then the rays travel another 20 cm. before they strike the screen, the displacement is—

$$\delta = \frac{e}{m}\,\frac{B\lambda}{v}\left(\frac{\lambda}{2} + L\right) = 1{\cdot}772 \times 10^7\,\frac{B.10}{v}\,(25)\text{ cm.}$$

If $V_1 = 10{,}000$ V, velocity of electrons $= 5{\cdot}95 \times 10^9$,

$$\text{and } \delta = \frac{1{\cdot}772 \times 10^7}{5{\cdot}95 \times 10^8}\,B \times 25 = 0{\cdot}745B\text{ cm.}$$

or approximately 7·5 mm. for unit strength of magnetic field.

If we assume that the field *B* is produced by two long coils (Fig. 69) one on each side of the tube and, say, 12 cm. long by 2·5 cm. wide, then if placed 2·5 cm. apart the field horizontal at the centre will be given by 0·32 $i\,n$ where i is the current in amperes and n = the number of turns in *each* coil. If $n = 1$, that is with a single turn, the value of B would be 0·32 with one ampere and the deflection would be about 2·5 mm.

If the current required to be studied were of the order of 0·1 A, *R.M.S.*, the number of turns would be 120 (this would give a deflection of 42·3 mm. on each

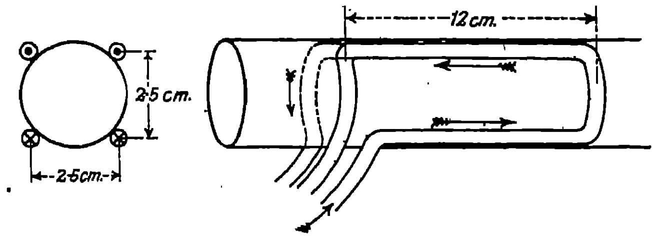


FIG. 69.—DIMENSIONS OF COILS PRODUCING ELECTRO-MAGNETIC FIELD IN THE BRAUN TUBE CONSIDERED IN THE TEXT

side of zero), and the value of the self induction would be about 3 millihenries. At a frequency of 1000 this would give a reactive voltage = 1·75 V ; at a frequency of 10,000 the voltage required would be 17·5 ; and at 100,000 frequency the voltage would be 175 V. The current sensitivity would be 1 mm. for about 0·003 A.

Case (*b*)—*Electrostatic Operation.* To take an example for the same oscillograph operating on the electrostatic forces—

If the speed of the corpuscles be the same as before, and if the electrostatic plates be 10 cm long and 1 cm. apart, i.e. $\lambda = 10$, $d = 1$, and if $L = 20$ as before, then : Displacement of spot under electrostatic field

$$= \delta = \frac{E}{v^2}\,\frac{e}{m}\,\frac{\lambda}{d}\left(L + \frac{\lambda}{2}\right)$$

where E, the voltage between the plates, is measured in electrostatic units.

If E be measured in volts, V—

$$\delta = \frac{V}{300}\frac{1}{v^2}\frac{e}{m}\frac{\lambda}{d}\left(L + \frac{\lambda}{2}\right).$$

If the voltage between cathode and anode $= 10{,}000$ V; $v = 5{\cdot}95 \times 10^9$; $e/m = 5{\cdot}316 \times 10^{17}$; $\lambda/d = 10$; and $\left(L + \frac{\lambda}{2}\right) = 25$. Therefore, $\delta = V \times {\cdot}0124$ or a deflection of 1 cm. for about 80 V or 3 cm. for a maximum voltage of 240.

As the capacity of two parallel plates, say, 2 cm. wide, 10 cm. long, and 1 cm. apart is of the order of $\frac{20}{4\pi}$ electrostatic units, or $2{\cdot}22 \times 10^{-12}$ farads or, say, 3×10^{-12} farads to allow for fringing and capacity of connecting wires, etc., this would take a current equal to 4·5 μA, at a R.M.S. voltage of 240 and a frequency of 1000 per sec. That is an apparent power of 1·08 milliwatts. On the other hand, electro-magnetic operation takes (apart from the power spent in the resistance of the coils) 0·1 A at 1·75 V when working at a frequency of 1000 per sec., or 0·175 watts, which is about 170 times the apparent power for electrostatic working. From this it will be seen how much more desirable it is to work the instrument with the electrostatic field when the power available is limited.

Upper Limit of Frequency. It has been claimed that there is no upper limit to the frequency that can be truly recorded by a cathode ray oscillograph, apart from the difficulty of obtaining a reliable record.

There is, however, an extreme upper limit depending on the speed of the electrons.

Thus, as far as the author knows, the extreme upper limit for a record of wave form is 220×10^6 cycles per sec., shown by Dufour using a stream of electrons with a velocity about 15×10^9 cm. per sec., and with a length of deflecting plate in the oscillograph about 8 cm.

The length of time a given electron takes to pass such a plate is $8/(15 \times 10^9) = 0{\cdot}5 \times 10^{-9}$ sec.; and the period of a *half-wave* of frequency 220×10^6 cycles per sec. is $1/(440 \times 10^6) = 2{\cdot}3 \times 10^{-9}$ sec.

If it were desired to study the wave form at this extreme frequency, the half-wave of the fifth harmonic would have a period equal to the time of flight of the electron past the plate, and its reduction in amplitude or attenuation would be in the ratio 2 to π of what it ought to be, since the electrons that enter between the plates where the pressure is zero will have a force increasing sinusoidally to a maximum at the middle of the plate and falling again to zero as they emerge, so that the acceleration is that due to the average value of the voltage and not that due to the maximum value. It is certain, however, that over ranges up to an absolute frequency of a million per sec., the magnification or diminution of a wave is negligible.

Details of Construction and Operation. It will be seen from the equations given on p. 130 that to increase the sensitivity of the cathode ray oscillograph it is necessary to diminish the speed of the moving electron; *or* to lengthen the duration of action of the field by lengthening the plates; *or* to increase the field strength by bringing the plates closer together.

If the cathode is cold it requires a fairly high initial voltage to start a stream of cathode rays. The voltage required depends on the distance apart of the electrodes, on the pressure of the residual gas in the tube, and also on the intensity of spot required on the screen or photographic plate. Generally speaking, the pressure is never less than 10,000 V, and voltages up to 60,000 are common.

As the velocity of the ray is proportional to the square root of the propelling voltage V and as the sensitivity of the oscillograph is inversely proportional to the velocity for magnetic deflection, and inversely proportional to the square of the velocity for electrostatic deflection, the oscillograph sensitivity is proportional to $1/\sqrt{V}$ in the magnetic, and to $1/V$ in the electrostatic case.

As the energy stored in each moving electron is proportional to the square of the velocity, the energy given up when it strikes the screen rises very rapidly with the increase of voltage. The screen S (Fig. 67) is covered with fine willemite crystals, or calcium tungstate, which fluoresce under the bombardment of the cathode rays, and thus indicate the position of the pencil rays or the "spot."

Johnson uses as the active material for the screen equal parts of calcium tungstate and zinc silicate, both specially prepared for fluorescence, with pure water glass as binder. He finds that this produces a generally more useful screen than either constituent alone. The pure tungstate gives a deep blue light which is about 30 times as active on the photographic plate as the yellow green light of the silicate, while the silicate gives a light which is many times brighter visually than that from the tungstate. By mixing the two a screen is produced more than half as bright visually as pure silicate and more than half as active photographically as pure calcium tungstate.

To increase the sensitivity of the oscillograph it is necessary to have a means of obtaining a stream of electrons under a low voltage. Sir Joseph Thomson has proposed a hot cathode of the same type as that used by Coolidge in his X-ray tube to produce free electrons. The cathode is generally in the form of a flat spiral surrounded by a cylindrical metallic sheath as shown in Fig. 70. This spiral can be made of tungsten or lime-coated platinum and is heated by passing a suitable current through it. The magnitude of the heating current is determined by the intensity of the cathode-anode stream, and the temperature must be such as to cause a sufficient supply of electrons for this stream. The object of the metallic sheath surrounding the heated spiral is to prevent spreading of the cathode stream as it leaves the spiral and to concentrate a larger portion of the stream on the anode.

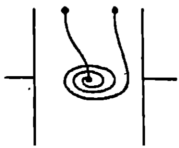


FIG. 70.—SPIRAL FOR HOT CATHODE RAY OSCILLOGRAPH

To obtain a fine spot on the screen it is necessary to adjust the value of the current in the filament, the voltage between the anode and cathode, and the pressure of the residual gas in the tube.

The spot is much smaller and more intense if there is a trace of gas left in the tube, a pressure of about $\frac{1}{1000}$ mm. of mercury being an average value. The value of the vacuum must, however, be governed by the appearance of the spot which should be very small, bright, and without blurs or halos. If the tube be made throughout of glass and all the connections hermetically sealed through the glass there is no ready means of altering the vacuum and, under working conditions, there is a tendency for the gas pressure to diminish and for the spot to become larger. It is preferable therefore, whenever possible, to have a connection from the tube to a suitable vacuum pump so that the vacuum can be adjusted to give the best spot.

It is also necessary, as will be seen later, to have a means of opening up the tube when it is desired to let the spot produced by the cathode beam fall directly on a photographic plate or film. In this case it is, of course, necessary to re-evacuate the tube after it has been opened to remove a record.

Alternative Forms of Records. When using the sealed-up tube such as shown in Fig. 67 there is no convenient method of allowing a plate or film to pass close to the screen and in addition the actinic power of the luminous spot is very small. It is necessary therefore, in order to obtain a record of the motion of the spot, to have some means of causing a displacement of the spot at right angles to the motion produced by the electrostatic or magnetic field being studied.

Thus, if it were possible to use one pair of the electrostatic plates to produce a field proportional to voltage being studied, and if the second pair of plates had a voltage applied to them which increased uniformly with time, then the curve produced on the screen would be a record of the voltage to be studied, plotted to a uniform time base. Similarly, if a magnetic field were arranged to produce a deflection of the cathode beam at right angles to the deflection produced by the voltage being studied, the record would be drawn to a uniform time base if the magnetic field increased uniformly with time.

When one is dealing with records from low frequency sources, say from a machine, it is quite easy to arrange a potentiometer to revolve synchronously with the wave to be examined and to supply either a current or a pressure varying uniformly with time. If the spot goes over the record time after time it produces a steady image which can be viewed directly, or it can be arranged to take a photograph of the trace by an ordinary camera and, as long as the phenomenon being studied is constant, the record on the screen will be constant and the photograph can be taken with a time exposure to suit the intensity of the spot on the screen. There are, however, many cases in which the frequency is too high or where no potentiometer is available, and it is then necessary to obtain a means of recording the curve.

In Fig. 67 when, say, an alternating potential is applied between the plates *P P*, the spot will be displaced up and down the screen as shown on the end view of the screen at (*a*). If now the coils *C C* carry a current of the same frequency as the supply to *P P* there will be a periodic displacement along the horizontal axis. As it is more convenient to have this periodic displacement simple harmonic motion, the current in the coils *C C* should be made as nearly sinusoidal as possible, and Fig. 71 shows an arrangement by which this result can be obtained. In this diagram *P P* represent the electrostatic plates of the oscillograph, and *C C* the coils which produce the horizontal displacement. A condenser K_2 is placed in parallel with *C C* to produce parallel resonance;

a condenser K_1 is connected with a variable inductance L to produce series resonance; and a non-inductive resistance R_2 is connected in series with L and K_1. Then, when R_2 is small, the current in the series circuit is very nearly a pure sine wave owing to the high impedance of L to all but the resonant frequency. This current is the resultant of the currents in C C and

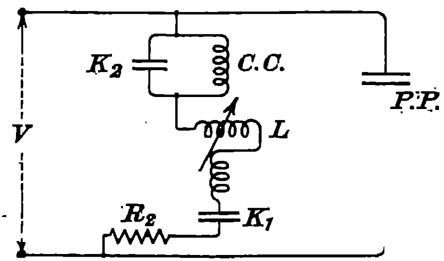


FIG. 71.—ARRANGEMENT OF CIRCUIT FOR OBTAINING SINUSOIDAL CURRENT IN COILS C C

K_2 and, as these are in resonance, the series current is small compared with the current in C C and in K_2.

The current in C C is still more nearly sinusoidal than the current in the series circuit since the impedance of the condenser K_2 varies inversely as the frequency and the impedance of C C directly as the frequency, and so the higher the harmonic, the greater the shunting effect of the condenser K_2 on the coils C C.

When L and K_1 are practically resonating, considerable changes in the phase of the currents can be made by varying the value L.

The resistance R_2 is placed in circuit to limit the value of the current until the circuits are adjusted.

The coils C C produce a horizontal displacement varying in simple harmonic motion, and the plates

P P a vertical movement proportional to the voltage, and since these motions differ practically 90 degrees in phase, the resultant trace is a circle or an ellipse when the voltage to be tested is sinusoidal. If the voltage be not sinusoidal an irregular curve will be drawn, perhaps as shown in Fig. 72. Since the displacement

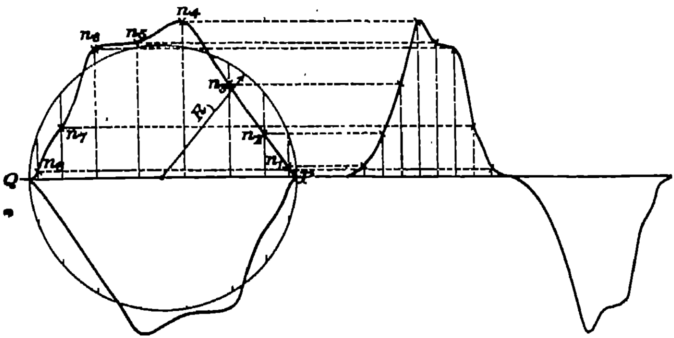


FIG. 72.—HYPOTHETICAL RECORD OBTAINED WITH A CATHODE RAY OSCILLOGRAPH

The curve is plotted to rectangular co-ordinates with time as base in the right-hand diagram

along *PQ* is simple harmonic, equal arcs measured round a circle, drawn with $R(= PQ/2)$, as radius, will represent equal periods of time, and the projections of these arcs on the line *PQ* will also represent equal times.

Perpendiculars erected at these points to meet the curve at n_1 n_2 n_3 . . . will give the values of the voltage at these equal time intervals, and if necessary the curve can then be redrawn to a uniform time base as shown on the right in Fig. 72. The number of the time intervals chosen should be greater, the more irregular the curve.

In some cases the displacement of the ray in a horizontal direction is not produced by a current or voltage that is in any simple relation to the vertical displacement, but as long as the horizontal displacement is in simple harmonic motion the record can always be redrawn. In this case, however, since the curve will not be retraced the exposure should preferably be short so as to record only one traverse of the spot across the plate.

Improved Cathode Ray Oscillographs. While the cathode ray oscillograph, as devised by Braun, opened up new fields of usefulness where oscillographs operating with mechanical systems even as light as a stretched wire would be inoperative, it is no doubt due to the valuable work of Dufour that the cathode ray oscillograph occupies its present unique position. As already mentioned, Dufour has, by means of improved technique and very elaborate apparatus, obtained photographic records of frequencies as high as 220,000,000 per sec. with a single trace, whereas a few years ago records of a million per sec. under such conditions would have been considered wonderful. This great improvement has been due to his recognition of the fact that, to work at high frequencies and obtain photographic records with a single trace over the plate, it is necessary that: (1) the cathode ray should fall directly on the plate; (2) there should be a small amount of residual gas in the tube, (for the reason explained on p. 155); (3) at the extremely high frequencies it is necessary to accelerate the cathode stream by a very high voltage even up to 60,000 V.

Once it is recognized that the problem must be faced of opening the tube to admit or withdraw a photographic plate, great modifications of Braun's original model become desirable.

As a pump must be provided capable of producing a high vacuum it is no longer necessary to have the joints in the apparatus hermetically sealed but a ground-in metal to metal joint sealed with grease is allowable, and it is possible to use metal instead of glass for a great portion of the "tube."

The Dufour Cathode Ray Oscillograph. Fig. 73 shows the cathode oscillograph as developed by Dufour. The lower portion of the apparatus is metallic and serves to contain the phosphorescent screen and the photographic apparatus. The part *v* is the intermediate portion of the apparatus and is where the electrostatic plates or the magnetic field is applied to the cathode stream. The upper portion, *d*, is the tube proper and contains at the upper end the cathode *e* and at the lower end a fine pin-hole tube *f* which acts as the anode and is connected to the metallic bell *a* forming the lower portion of the apparatus. The air-tight door *b* gives admission to the lower portion of the "tube," and allows the photographic plates and films to be taken in or out. The connection to the air pump is made at *m*.

Both *v* and *d* are made of glass and are arranged to fit into each and form an airtight joint. The tube *v* also fits into a coned hole in *a*. The tube *v* could be of metal if it were only required for internal electrostatic deflecting plates, but not if it is required to use it with external electrostatic plates or with a magnetic field. A conducting tube of any kind would be inadmissible for external plates and would be inadmissible

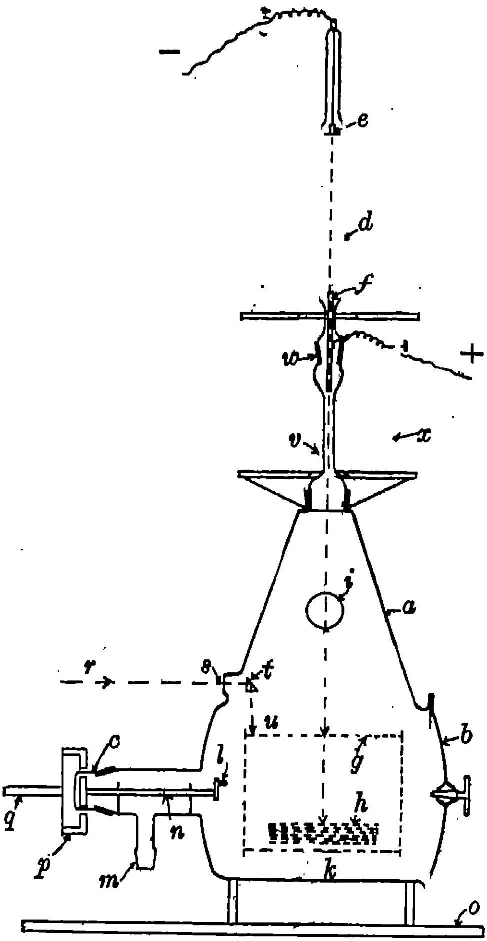


FIG. 73.—THE DUFOUR CATHODE RAY OSCILLOGRAPH

(*Note.*—This figure and Figs. 74–76 are reproduced by permission from *L'Oscillographe Cathodique, pour l'étude des basses, moyennes, et hautes frequences* by A. Dufour. (Etienne Chiron, 40 Rue de Seine, Paris; 9 francs).

at a frequency of, say, 50 cycles per sec. for an alternating magnetic field owing to the eddy currents produced.

When a glass tube is used with external plates the results are not so reliable as with internal plates, because the distribution of potential is not definite if the residual

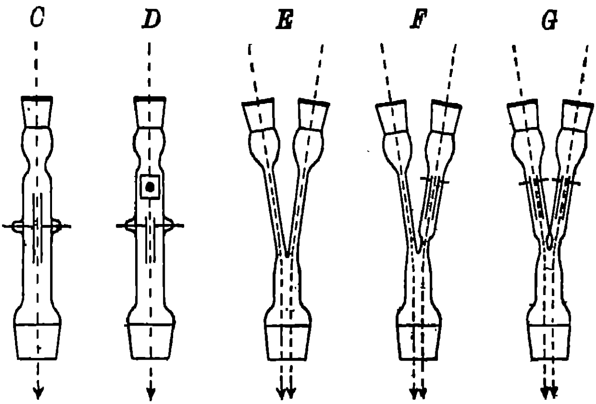


FIG. 74.—ALTERNATIVE FORMS OF TUBES TO SUIT DIFFERENT ARRANGEMENTS OF DEFLECTING PLATES, ETC., IN THE DUFOUR OSCILLOGRAPH

gas in the interior becomes ionized by the cathode stream. There will also be dielectric hysteresis and fatigue in the glass which will produce a distortion of the curve and also a displacement of phase. It is often convenient, however, to be able to use external plates or an external pair of deflecting coils at will.

The tube *v* as shown in Fig. 73 is used normally only with deflecting coils, and is replaced by the tube *C* (Fig. 74) when it is required to employ an electrostatic field, or by tube *D* when it is required to employ two electrostatic fields at right angles. The double tubes *E*, *F*, and *G* are arranged to enable simultaneous records to be obtained of two currents, *or* of a pressure and a current *or* of two pressures. The rays have to be made parallel to each by small electro-magnets operating where the two branches join the main tube.

In Fig. 73 is shown the arrangement Dufour employs to turn the photographic drum inside the vessel *a*. An internal shaft *n*, which can engage the drum *g* by means of the crank *l*, is driven through a positive magnetic clutch *p* from the outside shaft *q*. The lines of force from the magnet pass through a thin glass cap *c* which fits on to the extension of the vessel *a*.

The linear speed of the film on the drum can be made some 500 cm. per sec. and this is the arrangement used up to frequencies of the order of 8000, but for the higher of these frequencies their presence only is indicated and not their exact shape.

Dufour also has a high tension switch arranged to operate synchronously with the rotation of the outside shaft *q*. This is arranged to insure that the cathode ray will only be in action during one revolution of the drum, thus preventing fogging of the plates.

Fig. 75 shows a record obtained in this way, and is a trace of the current in the primary of an induction coil working in conjunction with a Wehnelt break.

When records of very high frequencies have to be obtained the speed of the drum is relatively so slow that no advantage is obtained by its use; plates are therefore used instead. It is then necessary to employ an auxiliary high frequency sinusoidal current, not necessarily or desirably of such high frequency as the source being investigated. This auxiliary current causes the cathode spot to traverse the plate on one axis, while the source being investigated causes the spot to traverse the plate on the other axis. The traverse due to the current being investigated is kept small compared with the auxiliary traverse.

If the auxiliary traverse be produced by a magnetic field due to, say, the current from a singing arc, and if

FIG. 75.—RECORD OF THE CURRENT IN THE PRIMARY OF AN INDUCTION COIL (WITH WEHNELT INTERRUPTER) OBTAINED BY MEANS OF A DUFOUR OSCILLOGRAPH

there be an auxiliary field at right angles to this produced by the application of a constant E.M.F. to the terminals of a coil, this second coil produces a trail or drift of the spot across the plate in the same axis as the source of E.M.F. being investigated, but at a rate slow compared with the movement of the spot due to the auxiliary source.

A curve obtained in this way is shown in Fig. 76. The traverse of the spot on the y axis is about 5 times as great as the traverse in the x axis, and the drift on the x axis during a complete cycle of the auxiliary field is about equal to the traverse of the source being investigated.

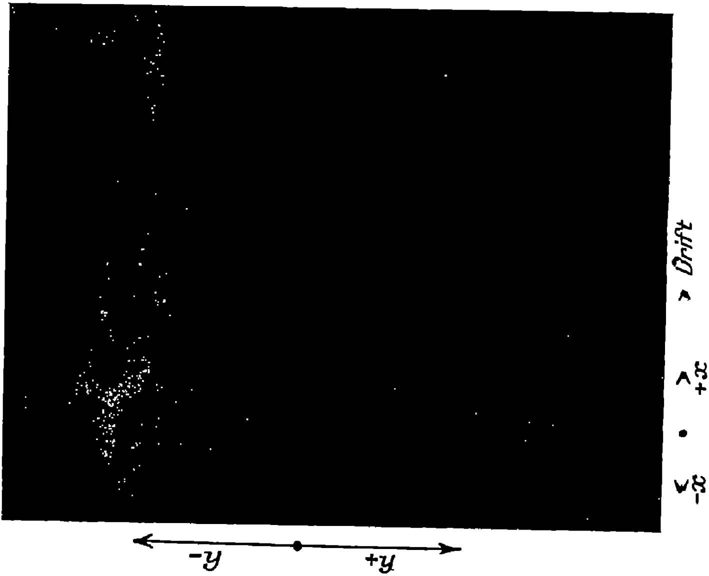


Fig. 76.—Record of Pressure Variations in a Singing Arc Obtained with a Dufour Oscillograph

This particular record (Fig. 76) is of the pressure variations of a singing arc. The auxiliary oscillations in the axis y are of a frequency of 10,000 per sec., the main oscillations in the x axis are of a frequency of about 240,000 per sec., and the high frequency oscillations superimposed on the main oscillations are of the order of 5,000,000 per sec.

It is sufficient to say that Dufour has achieved, as already mentioned, records up to a frequency of 220,000,000 per sec., but the records of these frequencies do not lend themselves so well for reproduction purposes. At such extreme frequencies the spot produced on the plate by the cathode ray moves at a rate of 800 to 900 kilometres per sec.

Hot Cathode Ray Oscillographs. Some crystals, such as tourmaline, have the property of giving an electric charge or quantity of electricity when subjected to mechanical pressure between two faces. This charge changes in sign when the pressure becomes negative, i.e. for a suction. The rise in electrical pressure due to the charge will, of course, depend on the capacity of the apparatus connected to the crystal and, as long as the capacity of the apparatus remains constant, the rise of electrical pressure is proportional to the rise of mechanical pressure.

This piezo electric effect gives a very valuable aid to the study of extremely high and rapidly varying mechanical pressures, and it has been so used by Sir Joseph Thomson, in conjunction with the cathode ray oscillograph, to study the rapid rise of pressure produced by explosions. To make the oscillograph more sensitive and to accelerate the electrons with a smaller voltage, Thomson used the hot cathode as already described. Dr. Keys, who collaborated with him, obtained in this way records of the pressure waves produced in water by the detonation of charges of guncotton and T.N.T.

The hot-cathode oscillograph as devised by Thomson and Keys was improved further in detail by Dr. A. B. Wood and it will be sufficient to illustrate the later instrument as representative of its class. Fig. 77 is a cross section showing the construction of the oscillograph designed by Dr. Wood and is self-explanatory to a large extent. It is a cathode ray oscillograph which has to be opened up to withdraw and insert the photographic plates and therefore is used in conjunction with an air pump. Near the top is a glass bulb containing both the cathode and the anode, which are comparatively close together as the instrument is designed for a working pressure of about 3000 V.

The electro-magnet, the poles of which embrace the tube, is used to produce the time-displacement of the cathode spot on the plate. This magnet is not used for the measurement of the electric current flowing as the eddy currents and hysteresis in the iron would introduce uncertainty, and so would the eddy currents in the brass tube itself. The measurements made are those due to the electrostatic forces and two pairs of plates at right angles are shown.

The records are obtained on photographic plates mounted on the six sides of the hexagonal plate holder shown at the bottom of the apparatus. This plate holder can be removed from the oscillograph when six records have been made.

The type of plate used is very important when one is dealing with cathode rays operating under low voltages, for the energy stored in the electron varies directly as the voltage; thus at a voltage of 3000 the energy will be one-twentieth of that at 60,000 V. To enable ordinary plates to be used successfully it is necessary to sprinkle them with a thin layer of calcium tungstate, but a better plan is to use Schumann plates which are specially prepared to have a minimum

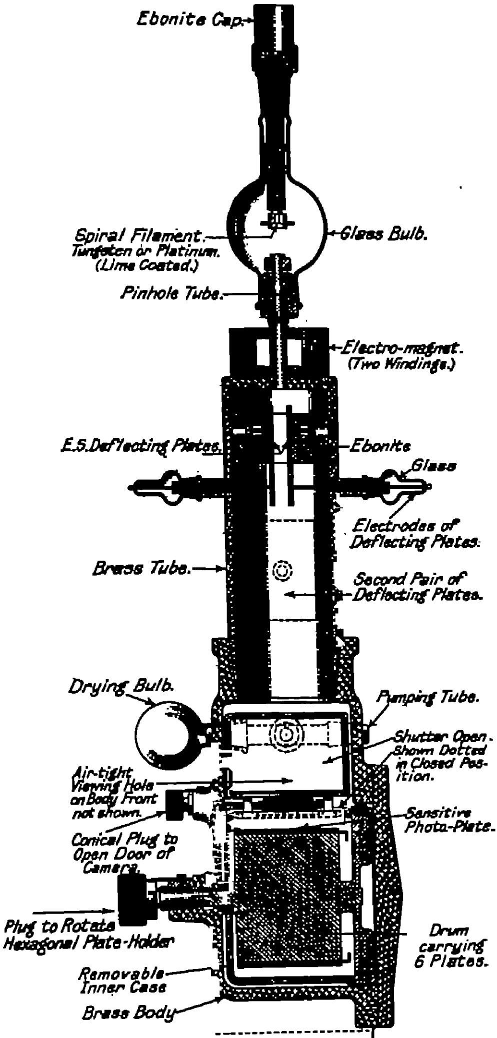
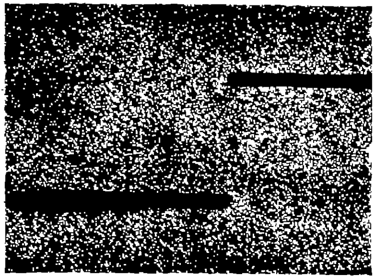


FIG. 77.—IMPROVED FORM OF HOT-CATHODE RAY OSCILLOGRAPH DUE TO DR. WOODS

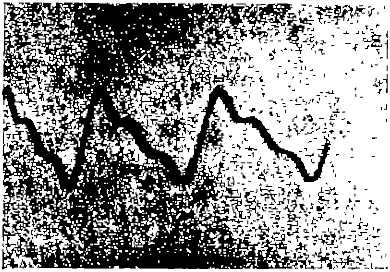
quantity of gelatine in the film. This type of plate allows more of the electrons to impinge directly on the silver granules and, as a result, a speed of spot relative to plate of 1000 metres per sec. is possible as compared with a speed of about 50 metres per sec. for ordinary gelatine plates. This compares with a speed of 800,000 to 900,000 metres per sec. claimed by Dufour for his oscillograph using a propelling pressure of 60,000 V.

It is evident therefore that if the results as regards clearness and arrangement were comparable, increasing the voltage twenty times would enable the relative velocity of spot and plate to be increased some 800 times; in other words as the voltage on the oscillograph is raised the photographic effects increase more rapidly than the energy stored in the electron. This is very fortunate as it enables records to be obtained at very high frequency as long as the propelling voltage of the rays is high.

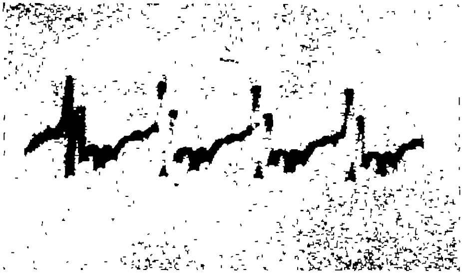
Characteristics of the Hot-Cathode Ray Oscillograph. The first essential test that it is necessary to apply to any oscillograph concerns its performance when a sudden change of voltage is applied to it. The record reproduced in Fig. 78(*a*) was obtained by suddenly applying 40 V to the electrostatic plates of a hot-cathode ray oscillograph when the spot was being trailed across the plate at a speed of about 5000 cm. per sec. As far as one is able to judge the change from one value to the other is absolutely instantaneous and there is no overshoot. Actually, there must be an extremely slight rounding of the curve owing to the finite speed of the electrons and there must be an overshoot if the electrostatic plates are connected to the source by leads having induction and comparatively



(*a*) Square wave test



(*b*) Frequency 16,000 per sec.



(*c*) Frequency 45,000 per sec.

Fig. 78.—Typical Records Obtained with a Hot-Cathode Oscillograph

low resistance. The first effect is so small that it would not appear, except at extremely high speed of spot relative to plate, the second effect is shown actually present in Fig. 76 given by Dufour.

It can be said, however, that even the slow speed electrons produced by voltages as low as 100 V would give records accurate up to frequencies as high as 1,000,000 per sec.

The curves shown in Fig. 78(*b*) and (*c*) are records of frequencies of the order of 16,000 per sec. and 45,000 per sec. respectively. All the records in Fig. 78 were kindly given the author, by Dr. Wood, and illustrate very well the clearness of the records obtained.

To obtain the time-variation of the spot across the plate Dr. Wood generally uses the electro magnet mentioned above (*see* Fig. 77), and places in series with it, a high inductance. If a constant E.M.F. be applied to this circuit, the rate of change of current, multiplied by the total self-induction of the circuit, must be equal to the applied E.M.F. just at the moment of switching on. It remains at approximately this value for a time equal to, say, a quarter of the time-constant of the circuit; if therefore, L/R be made large, the rate of change of current can be made constant for an appreciable time, and the spot can be trailed across the plate at a practically constant speed. To increase the speed of the spot it is only necessary to increase the applied E.M.F.

A similar result is attained if a condenser K be connected in parallel with one pair of electrostatic deflecting plates and the combination be charged through a resistance from a voltage V. The initial rate of rise of the voltage across the plates is V/KR and, as it is easy to make R large, the time constant KR can be made as long as necessary so that the rate of change of voltage remains practically constant over a considerable period, the actual rate of rise being varied by varying the applied voltage.

It is necessary, with this arrangement, to have a means of producing synchronism between the trailing of the spot across the screen and the occurrence of transient phenomena such as the record of the sudden switching-on of a voltage, as shown in Fig. 78(a). This is not difficult to arrange except at extreme speeds of travel of the spot relative to the plate.

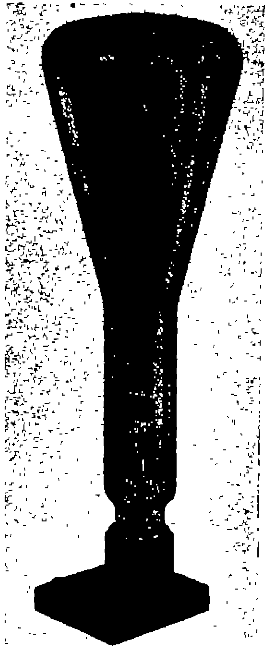


FIG. 79.—JOHNSON LOW-VOLTAGE CATHODE RAY OSCILLOGRAPH

Johnson Low-Voltage Cathode Ray Oscillograph. A new type of low-voltage cathode ray oscillograph has been developed by Johnson for the Western Electric Co. as shown in Fig. 79. It employs a closed glass tube, and, as in the original Braun tube, the record is made on a fluorescent screen and a photograph is taken of this trace which must remain steady for some seconds to allow for a time exposure with an ordinary camera.

The distinguishing features of this oscillograph are : (1) the low voltage, about 300 V, under which the electrons are propelled from the cathode anode ; (2) the closeness of the anode to the ca (3) the minute dimensions of these electrodes ; small amount of residual gas that can be] between the elec

To achieve (electrodes are into a small glas within the mai and the volume dual gas preser tween the electr not more than 1 (

All paths betwe electrodes are so and the pressure low that there a enough ions pres allow an arc to st

Fig. 80(*a*) show electrode unit w] mounted complet two sets of defl plates *P* at one the glass tube.

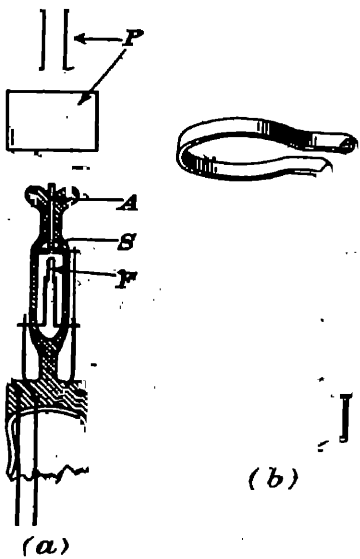


FIG. 80.—ELECTRODE UNIT (*a*) AND CATHODE (*b*), OF THE JOHNSON CATHODE RAY OSCILLOGRAPH

anode *A* is a platinum tube 1 cm. long by diameter and, as shown in the figure, this tube up almost to a screen *S* which is placed betwe cathode *F* and the anode *A*. The screen *S* h aperture at the centre, and is connected to the ca which is an oxide-coated platinum ribbon of the shown in Fig. 80(*b*). The particular shape o cathode is to protect it from the bombardment ions (positive) which pass through the aperture in the screen.

The cathode is heated by means of a current through it, and serves to provide a supply of free electrons. Those that pass near the aperture in the screen are drawn through by the strong electrostatic field between the anode and the screen, and are discharged in large part down the anode tube. Those of the electrons which have a flight along the axis of the tube emerge as a beam into the large tube and pass between the two pairs of plates.

The deflecting plates are of German silver to minimize eddy currents when a magnetic field is employed to produce the deflection.

Focusing in Cathode Ray Oscillographs. As the propelling voltage is low in the Johnson oscillograph, the velocity of the electrons is low compared with the values considered in the earlier parts of this chapter. There is, therefore, time for the electrostatic repulsion between the electrons to cause dispersal, and this would be serious except that it is possible to use the focusing due to the presence of residual gas in the tube. The lower the velocity of the electrons in the tube, the greater the amount of residual gas required, and, in the Johnson oscillograph a gas pressure of a few thousandths of a millimetre of mercury is used.

The use of residual gas was suggested by Dr. H. J. van der Bijl.

The focusing action of the residual gas can easily be explained and the same explanation applies also to the trace of gas left in the tube used by Dufour.

The number of electrons passing per sec. in a beam between the deflecting plates of a cathode ray oscillograph varies with the current flowing between the electrodes, and in the Johnson oscillograph it might be some 10^{14} electrons. The velocity of the electrons is about 10^9 cm. per sec., and therefore every centimetre measured along the beam contains about 10^5 electrons in flight.

Initially, this number would be concentrated in a beam of about 1 mm. diameter.

A number of these electrons in their flight will impinge on molecules of the residual gas and produce ionization, i.e. the molecule is broken up and the electron released. The release of the electron (carrying the negative charge) from the neutral molecule leaves the ion (carrying a positive charge), but the velocities of the original electron and of the displaced electron are high and they leave the path of the beam at a very high speed while the relatively heavy and slow ion leaves the beam at a slow speed.

There will therefore be along the beam an excess of positive ions over negative ions, and this means that the rays are drawn in towards the axis of the beam.

At first sight it might appear strange that any considerable percentage of the electrons could pass through the residual gas without coming into collision with the molecules.

At a pressure of 0·001 mm. of mercury the number of molecules present in the gas is still of the order of 3×10^{13} per cu. cm., and if the length of path in the oscillograph be 30 cm., the number in the path of the ray (assumed to be 1 sq. mm. in cross section) would be roughly 10^{13} molecules. The area of the shadows cast by the molecules (if one could assume a molecule to cast a shadow) would be more than the square millimetre which is the cross section of the beam and this would be so even when allowance was made for the attenuation of the gas in the beam due to the bombardment. The molecules are, however, in comparatively rapid motion or vibration and are not uniformly spaced so as to cut off the whole of the ray.

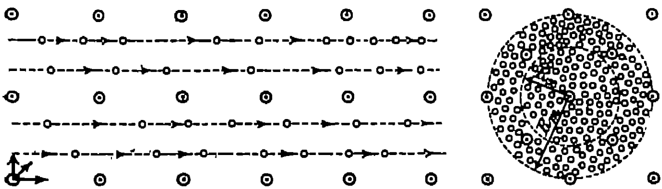


FIG. 81.—DIAGRAMMATIC REPRESENTATION OF ELECTRONS PASSING THROUGH RAREFIED GAS (NOT TO SCALE)

They are like so many shutters which, if placed edge to edge, could cut off the whole of a light source, but if allowed to oscillate past each other will allow light to pass.

The condition of affairs for a particular moment might be illustrated by Fig. 81, in which the large dots are supposed to represent molecules and the small dots electrons. The free space around a molecule may be of the order of a million times as great as the volume of the molecule itself, while the size of the electron is, in its turn, very small compared with the molecule. The number of electrons in a length of 1 cm. of the beam may be of the order of 10^5 and these are travelling down the beam with a velocity of about 10^9 cm. per sec.

If we imagine the ionized molecules (positively charged) to be uniformly distributed down the beam and over its cross section then the resultant charge could be represented, as far as its effect on other outside charges is concerned, by concentrating this charge along the axis of the beam, i.e. at its centre of gravity. In a similar way, the force due to the uniformly distributed electrons (negatively charged) could be considered to be due to these electrons concentrated along the axis.

The force on any individual electron on the boundary of the beam is towards the axis or away from the axis, depending on whether there is an excess of ions or of electrons.

For an electron inside the beam, at a distance R_x from the axis, the force is R_x/R_m of that at the boundary, and will act in the same direction as the force at the boundary if the distribution of electrons and ions is uniform.

If the numbers of electrons and ions are equal, there will be no force on the moving electrons, and the beam will only spread due to any initial divergence it may have. If the angle of divergence be known, then the number of free ions required per cm. length to turn the outside electrons and bring them to a focus at the screen can be estimated.

In any case, increasing the current increases the total ionization and the inward pull on the electrons and so brings the beam more quickly to a focus.

A characteristic of this and indeed of all cathode ray oscillographs is the conductivity between the deflecting plates, due to the ionization of the residual gas. This conductivity is not constant, but varies with the direction and magnitude of applied voltage; it is therefore not possible to use a series condenser to reduce the voltage applied to the instrument as this would produce appreciable distortion except when the frequency was very high. The conductivity between the plates depends on the number of free ions between them and will therefore increase with the size of the plates and their distance apart, provided that the cathode ray has been passing in the tube for some time before the deflecting voltage is applied to the plates. It will also increase with the pressure of the gas, if the cathode ray remain constant, as the number of collisions between electrons and molecules will increase, and so increase the number of free ions.

The current passing between the plates cannot be greater than that required to carry the free negative ions to the positive plate and the free positive ions to the negative plate as fast as they are produced by the bombardment. The current will therefore not increase proportionally to the voltage but, after a certain voltage is reached, will remain constant. This current is called the saturation current and will only start to increase again when the voltage between the deflecting plates is raised to such a point that it can produce ionization on its own account.

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